Calculations

This problem gives you the chance to:
• show understanding of calculations

1. Draw a circle around each calculation that has the same answer as $25 \div 2$.

Half of 25 $\qquad \frac{25}{2} \qquad 25 \div \frac{1}{2}$

$25 \times 0.5 \qquad 25 \times \frac{1}{2} \qquad 2 \div 25$

2. Which of the situations below have the same answer as $25 \div 2$?

Check (√) the correct ones.
Cross (**X**) the incorrect ones.

	√ or X
The cost in dollars each person pays if two people share the cost of a $25 meal.	
The number of miles traveled in two hours at 25 miles an hour.	
The amount in pounds each person gets when two pounds of candy is shared by 25 people.	
The weight in pounds of 25 parcels each weighing half a pound.	

8

Calculations		Rubric
The core elements of performance required by this task are: • show understanding of calculations Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives four correct answers with no extras: Half of 25 $\frac{25}{2}$ $25 \div \frac{1}{2}$ 25×0.5 $25 \times \frac{1}{2}$ $2 \div 25$ Subtract 1 point for each extra.	4 x 1	4
2. Gives correct answer: **Yes, No, No, Yes (√, X, X, √)**	4 x 1	4
Total Points		**8**

Calculations

Work the task and look at the rubric. What are the key mathematical ideas being assessed? What does a student need to understand to be successful on this task? ______________________________

____

____

____

Look at student work on part 1 finding equivalent expressions for 25÷2.
How many of your students:

- Answered all of 1 correctly?_________
- Omitted circles for:
  - Half of 25?________
  - 25/2?__________
  - 25 x 0.5?________
  - 25 x 1/2?_________
- Incorrectly circled:
  - 25÷1/2?_________
  - 2÷25?__________

What does this show you are about student understanding of number and operation? Were there any particular clusters that concerned you?

Now look at how students could make connections between number calculations and context. How many of your students marked:

Description of context for 25÷2	Correctly Marked	Incorrectly Marked
The cost in dollars each person pays if two people share the cost of a meal.	√	
The number of miles traveled in two hours at 25 miles an hour.	X	
The amount in pounds each person gets when two pounds of candy is shared by 25 people.	X	
The weight in pounds of 25 parcels each weighing half a pound.	√	

What characteristics of the context might have been problematic for students?
What is it about the operation of division is it that students are not understanding?
What experiences help students develop an understanding of the meaning of division?
What are the implications for instruction?

Looking at Student Work for Calculations

Most students did not show work or explain their thinking for this task. So here is just a report of work from the sample:

Failed to circle:	**%**
Half of 25	19%
25/2	25%
25 x 0.5	64%
25 x 1/2	71%

Incorrectly circled:	**%**
25÷1/2	42%
2÷25	33%

In matching 25÷2 to context students:

Description of context for 25÷2	Correctly Marked	Incorrectly Marked
The cost in dollars each person pays if two people share the cost of a meal.	√	17%
The number of miles traveled in two hours at 25 miles an hour.	**X**	23%
The amount in pounds each person gets when two pounds of candy is shared by 25 people.	**X**	42%
The weight in pounds of 25 parcels each weighing half a pound.	√	54%

5th Grade Task 5 Calculations

Student Task	Show understanding of division and multiplication calculations in problems and in context. Work with decimals and fractions.
Core Idea 1 Number Properties	**Understand numbers, ways of represent numbers, relationships among numbers, and number systems.** • Recognize and generate equivalent forms of commonly used fractions and decimals.
Core Idea 2 Number Operations	**Understand the meanings of operations and how they relate to each other, make reasonable estimates, and compute fluently.** • Develop fluency in dividing whole numbers. • Develop and use strategies to solve problems involving number operations with fractions and decimals.

Mathematics of the task:
- Recognize equivalent expressions in fractions and decimals
- Show understanding of the operation of division and multiple ways to represent it numerically
- Relate calculations and operations to context
- Work with division and multiplication of decimals and fractions and see the relationships between decimals and fractions
- Understand multiplication and division as inverse operations

Based on teacher observations, this is what fifth graders know and are able to do:
- Recognize that dividing by two is equivalent to taking half of a number
- Recognize that dividing by two is the same as a number of two or written as a fraction with a denominator of two
- Division as a sharing into equal parts
- Division as a method for finding a rate (miles per hour)

Areas of difficulty for fifth graders:
- Seeing equivalence between dividing by 2 and multiplying by 0.5
- Seeing equivalence between dividing by 2 and multiplying by 1/2
- Seeing that dividing by 2 is not equivalent to dividing by 1/2
- Seeing that order matters when using division notation, e.g. $2 \div 25 \neq 25 \div 2$

MARS Test Task 5 Frequency Distribution and Bar Graph, Grade 5

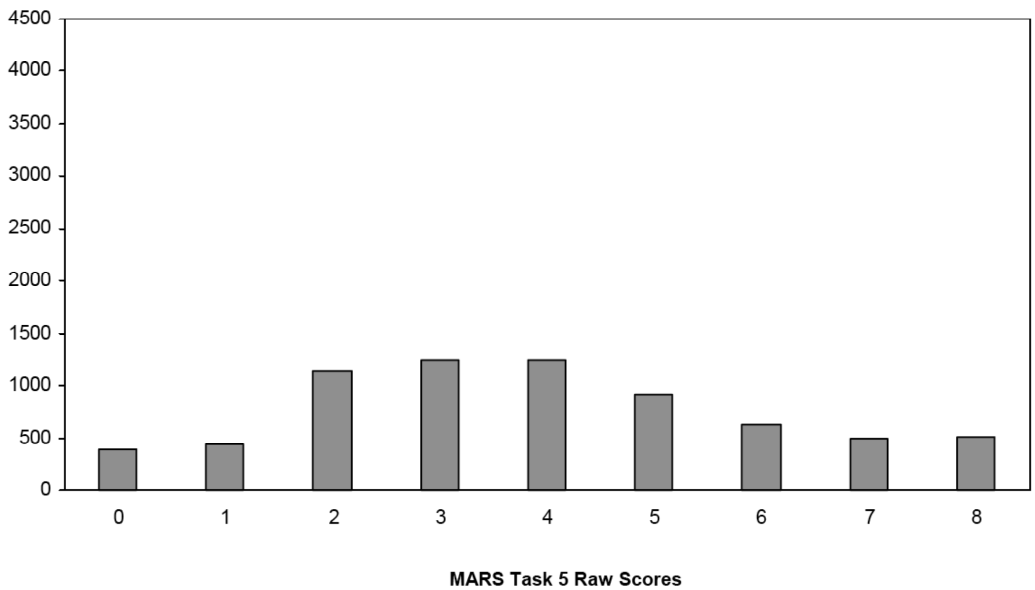
Task 5 - Calculations

Mean: 3.89 StdDev: 2.13

Table 29: Frequency Distribution of MARS Test Task 5, Grade 5

Task 5 Scores	Student Count	% at or below	% at or above
0	394	5.6%	100.0%
1	447	12.0%	94.4%
2	1136	28.2%	88.0%
3	1248	46.0%	71.8%
4	1245	63.7%	54.0%
5	920	76.9%	36.3%
6	634	85.9%	23.1%
7	487	92.8%	14.1%
8	502	100.0%	7.2%

Figure 38: Bar Graph of MARS Test Task 5 Raw Scores, Grade 5



The maximum score available for this task is 8 points.
The minimum score needed for a level three response, meeting standard, is 5 points.

Most students, 88%, could recognize that dividing by 2 is equivalent to taking half of a number and that sharing is equivalent to division. About half the students, 54%, knew that dividing by 2 could be written as a fraction in halves and that dividing could produce a rate. Some students, 36%, could also recognize that sharing 2 pounds by 25 people is not equal to 25÷2. About 7% of the students could meet all the demands of the task including seeing that multiplying by 1/2 or 0.5 is equivalent to dividing by 2 and finding contexts that represent and don't represent the calculation 25÷2. Almost 6% of the students scored no points on this task. 60% of the students with this score attempted the task.

Calculations

Points	Understandings	Misunderstandings
0	60% of the students with this score attempted the task.	Students did not recognize that half of a number is equal to dividing by 2 and/or thought that dividing by 2 is the same as dividing by 1/2.
2	Students could recognize that dividing by 2 is the same as taking half of a number and that share 25 items between 2 people can be solved by dividing by 2.	Students struggled with recognizing that dividing by 2 can also be written as a fraction with a denominator of 2. Students also struggled with recognizing that the rate for 2 hours can be written as a calculation dividing by 2.
4	Students could see that 25÷2 is equivalent to half of 25 and 25/2. Students could see that the 25÷2 is equivalent to sharing 25 between 2 people but not equivalent to finding the miles traveled in 2 hours at 25 miles per hour.	*See table for more details.*
5	Students could see that 25÷2 is also equivalent to 25 x 0.5.	*See table for more details.*
8	Students could recognize equivalent expressions for 25÷2 in other calculations including expressions with decimals and fractions and in context. Students could also find expressions ad context that were not equivalent.	

Implications for Instruction

Teaching students procedures is not enough. Students need discussion and experiences to help them deepen their understanding of operation. As students work with new concepts such as decimals and fractions they need to make connections to what they know about working with whole numbers and see how the operation effects results. Previous notions that dividing gives a smaller answer are now tested and no longer hold true. Students should be experimenting with these conjectures and talking about results with classmates.

Some students struggle with just the various notations used for division and how the order of language does not follow the same syntax as the spoken language. Twenty-five divided by two sounds the same as 25÷2, but often the classroom language says 2 goes into 25 leading them to think this is the same as 2÷25.

When deepening their understanding of operation with new numbers, students need to work with the meaning in context. What is different about the context? How many different ways are there to write a division problem? When are multiplying and division the same ($1/2 \times 25 = 25 \div 2$)? When are they different ($25 \div 2 \neq 25 \div 1/2$)?

Ideas for Action Research

The Role of Models and Context in Understanding Division

Some students at this grade level are still having trouble understanding the operation of division. These students need more experience with the types of actions that call for division action. With your colleagues, study the diagram below. What types of division problem-types do you think that students understand? What types of division problem-types do students need more help with? Can you design a series of problems or classroom lessons to examine different division situations?

Problem Type	*Multiplication*	*Measurement Division*	*Partitive Division*
Grouping/ Partitioning	Gene has 4 tomato plants. There are 6 tomatoes on each plant. How many tomatoes are there all together?	Gene has some tomato plants. There are 6 tomatoes on each plant. All together there are 24 tomatoes. How many tomato plants does Gene have?	Gene has 4 tomato plants. There are the same number of tomatoes on each plant. All together there are 20 tomatoes. How many tomatoes are there on each tomato plant?
Rate	Ellen walks 3 miles an hour. How many miles does she walk in 5 hours?	Ellen walks 3 miles an hour. How many hours will it take her to walk 15 miles?	Ellen walked 15 miles. It took her 5 hours. If she walked the same speed the whole way, how far did she walk in one hour?
Price	Pies cost 4 dollars each. How much do 7 pies cost?	Pies cost 4 dollars each. How many pies can you buy for $28?	Jan bought 7 pies. He spent a total of $28. If each pie cost the same amount, how much did one pie cost?
Multiplicative Comparison	The giraffe in the zoo is 3 times as tall as the kangaroo. The kangaroo is 6 feet tall. How tall is the giraffe?	The giraffe is 18 feet tall. The kangaroo is 6 feet tall. The giraffe is how many times taller than the kangaroo?	The giraffe is 18 feet tall. She is 3 times as tall as the kangaroo. How tall is the kangaroo?

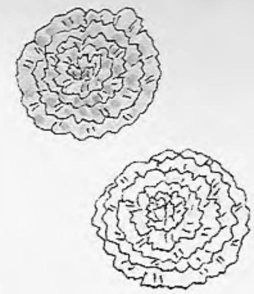
FIGURE 4.7 *Grouping/Partitioning, Rate, Price, and Multiplicative Comparison Problems*

from Children's Mathematics, Cognitively Guided Instruction *by Heinemann Press*

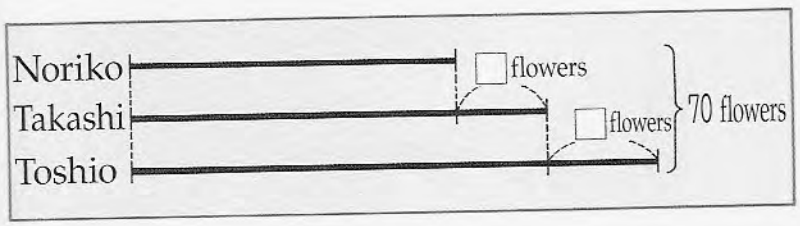
Models are also useful in helping students visualize the process or action of division. Study the models below. How do they help clarify division?

Model #1

① Noriko, Takashi and Toshio are making 70 paper flowers together. Takashi will make 5 more flowers than Noriko. Toshio will make 6 more flowers than Takashi.



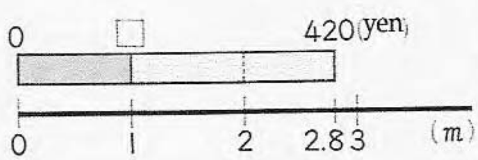
How many paper flowers will they each make?

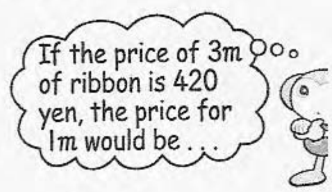


Model #2

1 Dividing by a Decimal Number

1 When we bought 2.8m of ribbon, the price was 420 yen. How much was the price of 1m of ribbon?





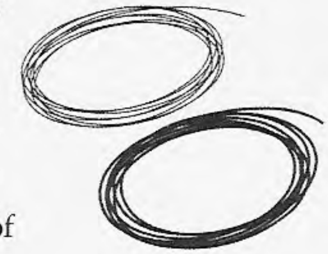
How should we write the math sentence?

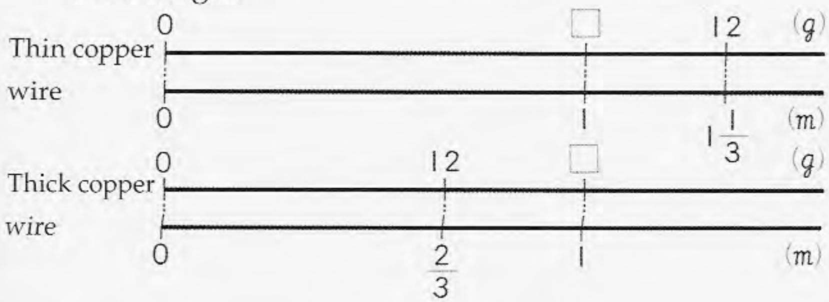
Price we paid ÷ Length we bought (Unit: m) = Price for 1m

Model #3

▶ **Divisors and the size of the quotient**

5 $1\frac{1}{3}$ m of thin copper wire weighs 12g and $\frac{2}{3}$ m of thick copper wire weighs 12g. How many g does 1m of each kind of wire weigh?





1 Please write math sentences and find the answers.

$12 \div 1\frac{1}{3} = \square$ $\quad 12 \div \frac{2}{3} = \square$

Let's investigate the relationship between the divisor and the size of the quotient!

2 Of the two math sentences above, which one will have a quotient that is greater than the dividend 12?

Even when dividing with fractions, if you divide by a number less than 1, the quotient will be greater than the dividend.

from Japanese series: Mathematics for Elementary School available from Global Education Resources in Madison, New Jersey

Now think about the actions in the task Division. What models could you make to illustrate each situation?

Ideas for Action Research 2

Another area for investigation is to get students making conjectures about multiplying and dividing. Students can learn to start making simple proofs at this stage using models and by giving examples. They can also disprove ideas by providing counter examples. In the book Thinking Mathematically by Carpenter, Franke, and Levi there are great examples about students generating conjectures about the properties of addition and multiplication with video of some students mathematical arguments.

This might be a starting place to work with colleagues for developing some lessons about operation of division.

If you look at the 8th Mars task 2008 Multiples of 10, you can see a format that might help start to develop the classroom list of conjectures. See below:

1. Adam says, "If you add together two multiples of 10 you get a multiple of 20."

   a. Give an example to show that this can be true. ______________________________

   b. Give an example to show that this is not always true. ______________________________

2. Eli says, "If you multiply two multiples of 10 you get a multiple of 100."

   a. Give an example to show that this can be true. ______________________________

   b. Explain why this is always true. ______________________________

____

____

How might you reword these examples to explore some of the common misconceptions that you have seen in this task?

What are other ways to introduce some investigations of these important ideas?

Reflecting on the Results for Fifth Grade as a Whole:

Think about student work through the collection of tasks and the implications for instruction. What are some of the big misconceptions or difficulties that really hit home for you?

If you were to describe one or two big ideas to take away and use for planning for next year, what would they be?

What were some of the qualities that you saw in good work or strategies used by good students that you would like to help other students develop?

Five areas stood out for the Collaborative as a whole. These include:

1. **<u>Reading word problems to identify constraints and problem-solving strategies:</u>** Students had a difficult time reading and interpreting constraints. In Table Decorations students struggled with identifying what is given or known what is missing. Do I know the part or the whole? They also had trouble identifying the meaning of the answers to calculations because they didn't use problem-solving tools like labels to help them track their thinking. When trying to compare the data sets in bird survey students had difficulty with identifying which attributes should be compared. what is it I am trying to find out? In Floors 4U students also had trouble with the constraints. What is the area that needs to stay the same? What does it mean to make a different example? What am I trying to find out? More than 24% did not attempt to find the perimeter.
2. **<u>Understanding the operation of division and multiplication:</u>** In Table Decorations students sometimes had difficulty choosing the correct operation confusing when to multiply and divide or using drawing and counting or repeated addition and subtraction because they are not confident with multiplication and division. In Calculations students confused multiplying and division or had trouble recognizing equivalent expressions between the two. Students could not recognize division in context.
3. **<u>Understanding scale and rate:</u>** In Helter Skelter students had difficulty with scale and rate. Part of this was due to a misunderstanding of elapsed time versus total time. In Table Decorations students could think about rate or proportional amounts with small numbers but weren't comfortable enough with the process to continue the same reasoning as the numbers got larger. In Calculations students could not reason about multiplication and division when the numbers were small (less than 1). They had difficulty with rates in context.
4. **<u>Composing and Decomposing Shapes:</u>** Many students could reason about area and perimeter in a specific context, but not expand those ideas to generating their own examples. They had trouble reasoning about the dimensions and area of triangle within in square, because they could decompose the figure into know parts or they generalized ideas about rectangles to triangles.

How Are Fifth Grade Students Succeeding on the Ramps?

Examining the Ramp: Looking at Responses of the Early 4's (30,31,32,33)
With a group of colleagues look at student work around 29 – 32 points. Use the papers provided or pick some from your own students.

How are students performing on the ramp?
What things impressed you about their performance?
What are skills or ideas they still need to work on?
Are students relying on previous arithmetic skills rather than moving up to more grade level strategies?
What was missing that you would hope to see from students working at this level?
How do you help students at this level step up their performance or see a standard to aim for in explaining their thinking?
Are our expectations high enough to these students?

How do we provide models to help these students see how their work can be improved or what they are striving for?
Do you think errors were caused by lack of exposure to ideas or misconceptions?
What would a student need to fix or correct their errors?
What is missing to make it a top-notch response?
What concerns you about their work?
What strategies did you see that might be useful to show to the whole class?

Table Decorations

- Using a proportion with large numbers
- Explaining how to expand the rate to large numbers

The Helter-Skelter

- Distinguishing between elapsed time and total time when drawing a graph
- Interpreting relationships between measures on the time distance graph to find how long she was above 10 yards

Bird Survey

- Comparing and contrasting relevant data on the graph to find which state Jody visited
- Being specific about how many/ how much less

Floors 4U

- Composing and decomposing a shape
- Interpreting a diagram
- Finding area of a triangle

Calculations

- Recognizing that multiplying by 0.5 or 1/2 is the same as dividing by 2

Scoring notes: For almost all the papers below I think the explanation for Bird Survey is incorrect. Most responses don't tell which birds or how many birds are alike or explain how or why or what about the other states eliminates them as possibilities.

Tim – 30 points

4. Last week, Flora used 100 flowers to complete all of her table decorations.

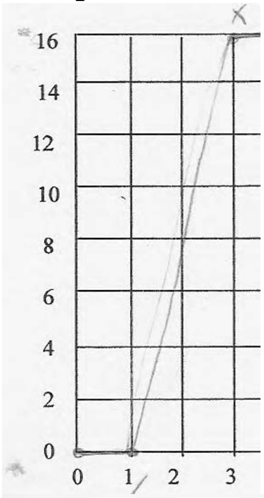
How many of these flowers were roses? 40 roses

How many table decorations did she make? 20 decorations
Explain how you figured this out.

0020
5)100
-100
000

I devided 100, the number of floweres, by 5, the number of flowers in one decoration. I got 20 decorad'ins.

Tim, part 2



2. For how long is Bridie more than 10 yards above the ground? ___1___ minutes

Do you think Jody went to Massachusetts, Hawaii or Arizona?

Arizona

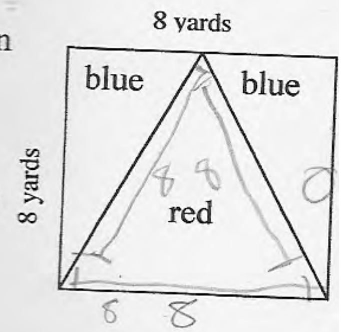
Explain clearly how you decided.

I compaired Jody's chart with Randy, Lynda and Alan's charts. Jody's chart was the most similar to Randy's chart so she must have gone to Arizona.

10

2. The leisure center wants the carpet for this square floor to be in two different colors like the diagram.

The floor is 8 yards long and 8 yards wide.
How much red carpet will be needed? ___24___ square yards



Explain how you figured this out.

I figured out that all the sides of the triangle are 8 yards long so 24 square yards of carpet will be needed

Tim part 3

25×0.5 X $25 \times \frac{1}{2}$ X

X X

Heidi – 31 points

4. Last week, Flora used 100 flowers to complete all of her table decorations.

How many of these flowers were roses? 50 roses XX

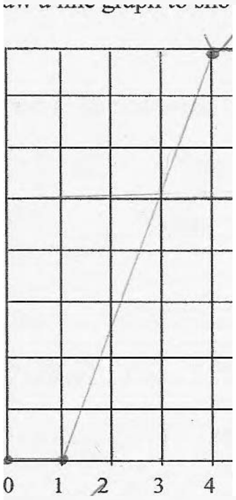
$2\overline{)100}$ = 50, −100, 0

How many table decorations did she make? 20 table decorations ✓✓

Explain how you figured this out.

$5\overline{)100}$ = 20, −10, 00, −00, 0 ✓

Since there is 5 flowers in each vase, I divided one hundred by five, and got twenty. ✓ (5 can go into ten 2 times. 10 mius 10 is zero. Bring down your zero from the 100 and get zero. 5 can go in to zero, zero times. Zero minus zero, is zero. Your quotient is 20.)



2. For how long is Bridie more than 10 yards above the ground? 3 minutes

Heidi part 2

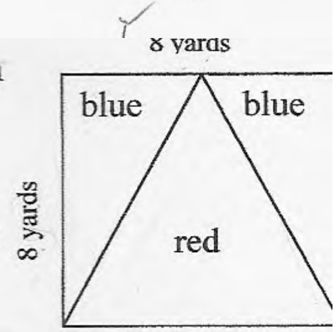
Do you think Jody went to Massachusetts, Hawaii or Arizona?

Arizona

Explain clearly how you decided.

I think Jody went to Arizona because when Randy went to Arizona, he saw the most of the same birds that Jody saw.

2. The leisure center wants the carpet for this square floor to be in two different colors like the diagram.



The floor is 8 yards long and 8 yards wide.
How much red carpet will be needed? 64 square yards

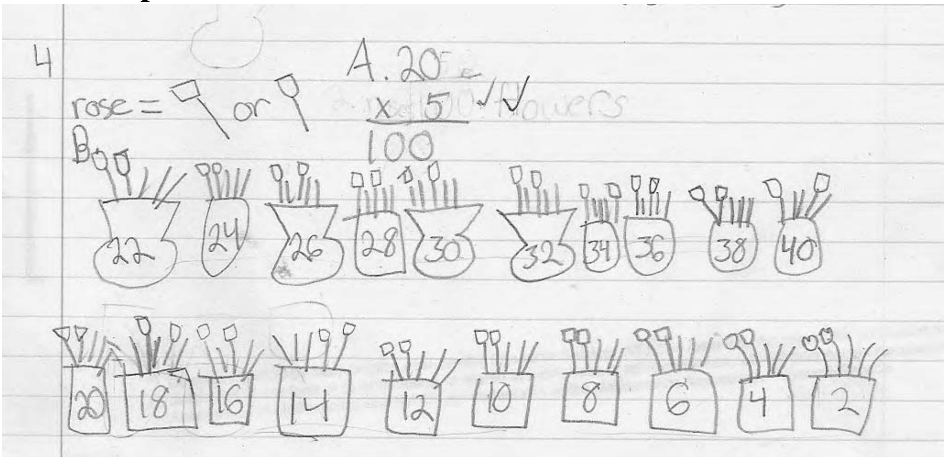
Explain how you figured this out.

One side of the perimeter is 8 yards. 8 times 8 equals 64. 64 is the area.

25×0.5 $\qquad$ $25 \times \frac{1}{2}$

$$\begin{array}{r} {}^{2}25 \\ \times\ 0.5 \\ \hline 12.5 \end{array}$$

Nina – 31 points



Nina part 2

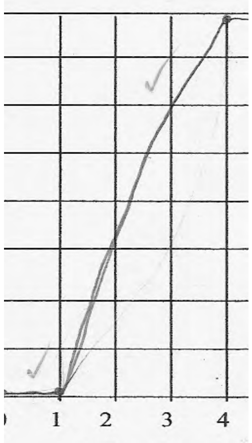
4. Last week, Flora used 100 flowers to complete all of her table decorations.

How many of these flowers were roses? 40 roses ✓ ✓ 1 1

How many table decorations did she make? 20 decorations ✓ 1 1
Explain how you figured this out.

I figured this out by drawing 20 vases & filling them with 5 flowers each. Then, I drew 2 roses for each vases. After that, I counted each rose & came up with forty roses. (continued on extra sheet) 1 ✓ 8 8 **8**

Copyright © 2010 by Mathematics Assessment Page Table Decorations



2. For how long is Bridie more than 10 yards above the ground? 1 ½ minutes ✗

✗ 0

Nina part 3

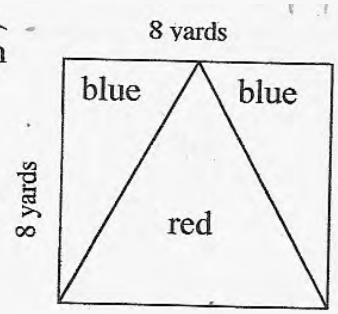
Do you think Jody went to Massachusetts, Hawaii or Arizona? Arizona ✓

Explain clearly how you decided.

I decided this because she has similar birds to Randys birds. ✓

them to get the area = 24. ✗

2. The leisure center wants the carpet for this square floor to be in two different colors like the diagram.



The floor is 8 yards long and 8 yards wide. ✗ $8 \times 8 = 64$ ✗
How much red carpet will be needed? 64 square yards

Explain how you figured this out.

I figured it out by taking the height, 8 yards and multipy it by the base, 8 yards. After multiplying it, I got 64 yards^2.

✓ ✓

25 × 0.5

✓ ✓

$25 \times \frac{1}{2}$

Austin – 32 points

5 combinations in each so I multiplied 2 and 3 by 15 and added them up.

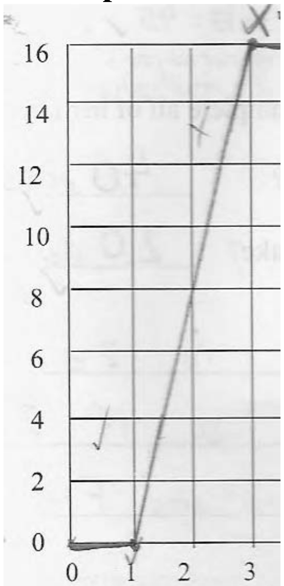
4. Last week, Flora used 100 flowers to complete all of her table decorations.

How many of these flowers were roses? 40 roses ✓ $20 \times 2 = 40$ (# of decoration, how many roses in each one) ||

How many table decorations did she make? 20 decorations ✓ ||
Explain how you figured this out.

I know 1 decoration is 2·3, and there are 5 flowers in all so if I divide 100 by 5 I get 20 decorations. Flora made 20 decorations. ✓ ||

Austin part 2



2. For how long is Bridie more than 10 yards above the ground? 2 minutes

□ = 1 minute + ⏢ + △ = 1 minute

Do you think Jody went to Massachusetts, Hawaii or Arizona? Arizona

Explain clearly how you decided.

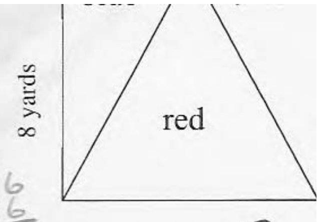
I decided this because Jody and Randy had the most similar birds. They had More birds in common

10

Austin part 3

The floor is 8 yards long and 8 yards wide.
How much red carpet will be needed? 32 square yards

Explain how you figured this out.



$$4\overline{)64} = 16 \qquad 16 + 16 = 32$$

2

I know that 8x8 =64 so that is the area of the square. I then visalized the red triangle to be split in half to equal the blues. I then got 4 piecses and divided 64 by 4 and go 16. But 16 was only half of the red so I added 16 +16 and got 32. The red carpet is 32 square yd.

Page 8

Floors 4 U

25×0.5

$25 \times \frac{1}{2}$

Santino – 32 points

4. Last week, Flora used 100 flowers to complete all of her table decorations.

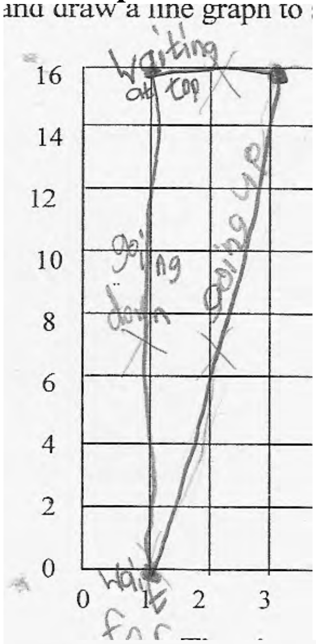
How many of these flowers were roses? 40

How many table decorations did she make? 20
Explain how you figured this out.

There are 5 flowers for every decoration. There are 100 flowers. I divided 100 ÷ 5 which equals 20.

Santino part 2

and draw a line graph to



2. For how long is Bridie more than 10 yards above the ground? 4 minutes

Sparrow	2

Do you think Jody went to Massachusetts, Hawaii or Arizona? Arizona

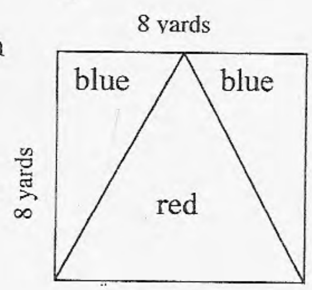
Explain clearly how you decided.

She saw most of the birds Randy saw.

2. The leisure center wants the carpet for this square floor to be in two different colors like the diagram.

The floor is 8 yards long and 8 yards wide.
How much red carpet will be needed? 32 square yards

Explain how you figured this out.



8x8 equals 64. Half of the carpet is red, so half of 64 equals 32

Santino part 3

25×0.5

$$\begin{array}{r} ^{2}25 \\ \times\ 5 \\ \hline 125 \end{array}$$

$25 \times \frac{1}{2}$

Mondo 33 points

4. Last week, Flora used 100 flowers to complete all of her table decorations.

How many of these flowers were roses? 40

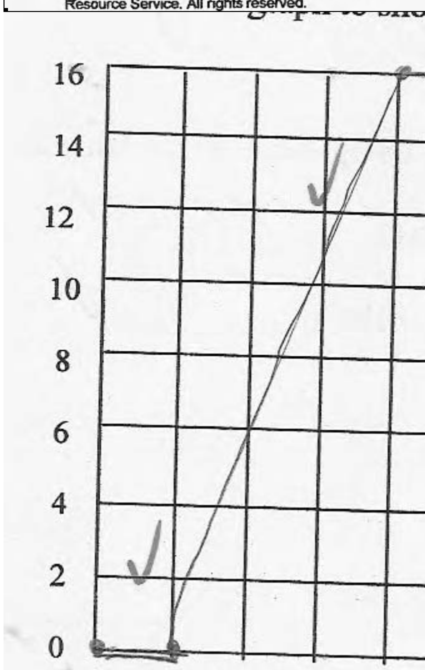
How many table decorations did she make? 20

Explain how you figured this out.

I tryed every Combination until I got to 20 and it equaled 100. Trial and error.

$$\begin{array}{r} 20 \\ \times\ 2 \\ \hline 40 \end{array} \qquad \begin{array}{r} 20 \\ \times\ 3 \\ \hline 60 \end{array}$$

Page 8 Table Decorations



Mondo part 2

2. For how long is Bridie more than 10 yards above the ground? ___\ ___minutes

Do you think Jody went to Massachusetts, Hawaii or Arizona? Arizona

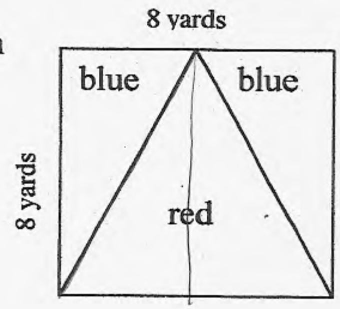
Explain clearly how you decided.

I think Jody went to Arizona because they saw a lot of the same birds and close to the same amount of each bird.

STOP

STOP 10

2. The leisure center wants the carpet for this square floor to be in two different colors like the diagram.



The floor is 8 yards long and 8 yards wide.
How much red carpet will be needed? __X__square yards

Explain how you figured this out.

Blue + Blue = Red. I multiplyed 8x8=64 then divided 64÷2=8 becouse Red is half of the area.

$8 \times 8 = 64$

$64 \div 2 = 8$

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Floors 4 LI

25×0.5

$25 \times \frac{1}{2}$