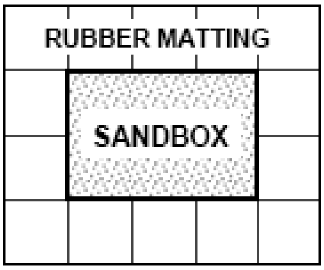
In the Playground

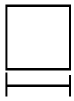
This problem gives you the chance to:
• work with areas

The playground committee decides to make a sandbox area for toddlers.
For safety reasons, the sandbox must be surrounded by a strip of rubber matting that is 2 feet wide.

Here is a scale drawing of the sandbox.



SCALE:



= 4 square feet

2 feet

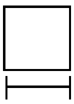
1. Find the area of the sandbox and the area of the rubber matting.

   Sandbox area: ___________ square feet Rubber matting area: ___________ square feet

More children are using the playground, so the committee decides to double the area of the sandbox.

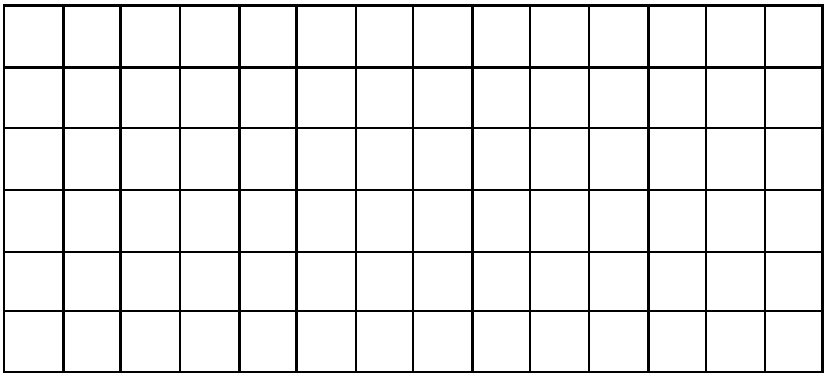
2. Design a new rectangular sandbox that has double the area of the original sandbox.
   On the grid below, make a scale drawing of the new sandbox and the surrounding rubber matting.

SCALE:



= 4 square feet

2 feet



3. How many square feet of rubber matting will they need? ____________________square feet

4. What is the length and width of the new sandbox? length __________ feet

   width __________ feet

8

In the Playground **Grade 6** Rubric		
The core elements of performance required by this task are: • work with areas Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answers: **24** square feet (Accept 80 square feet) **56** square feet	1 1	2
2. Draws a correct diagram: Rectangular area of sand, 48 square feet (12 squares) Surrounded by a row of squares of "rubber matting"	2 1	3
3. Gives correct answer: their number of squares **x 4** (dependent on their diagram and following the one row rule) **80** or **72** (accept 120)	1ft	1
4. Gives a correct answer dependent on their diagram for question 2: **12** and **4** or **8** and **6** or **2 and 24**	2x1ft	2
Total Points		**8**

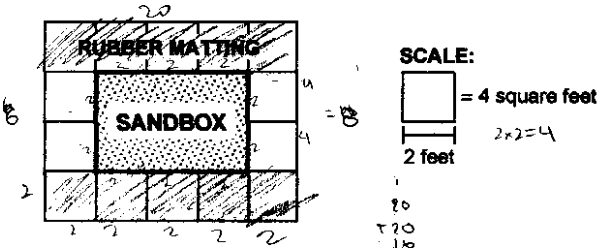
Looking at Student Work on In the Playground

Student A highlights the dimensions by labeling the sides of the sandbox and matting with two's. The student seems to use a counting by 4's strategy to find the area of the rubber matting in part 1. Student A is able to use the information from part 1 to design a new sandbox with twice the area of the original. The student can think about the scale for linear and area measures.

Student A

The playground committee decides to make a sandbox area for toddlers.
For safety reasons, the sandbox must be surrounded by a strip of rubber matting that is 2 feet wide.

Here is a scale drawing of the sandbox.

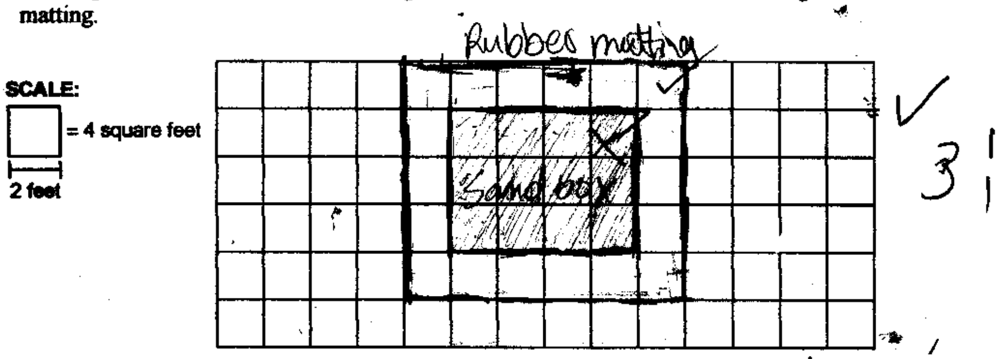


1. **Find the area of the sandbox and the area of the rubber matting.**

   Sandbox area: 24 **square feet** **Rubber matting area:** 56 **square feet**

More children are using the playground, so the committee decides to double the area of the sandbox.

2. **Design a new rectangular sandbox that has double the area of the original sandbox. On the grid below, make a scale drawing of the new sandbox and the surrounding rubber matting.**



3. **How many square feet of rubber matting will they need?** 72 **square**

4. **What is the length and width of the new sandbox?** **length** 8 **feet**

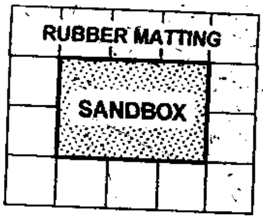
   width 6 **feet**

Student B uses an interesting technique to find the area of the matting. The student finds the area of the total play structure and subtracts the area of the sandbox. It seems most likely that the student found the area of each by multiplying the dimensions together, because the student is able to reason correctly about the linear dimensions in part 4. However, the student struggles with the area of the new matting, multiplying the number of squares by the linear scale of 2 instead of by the area scale of 4.

Student B

The playground committee decides to make a sandbox area for toddlers.
For safety reasons, the sandbox must be surrounded by a strip of rubber matting that is 2 feet wide.

Here is a scale drawing of the sandbox.



SCALE: □ = 4 square feet
2 feet

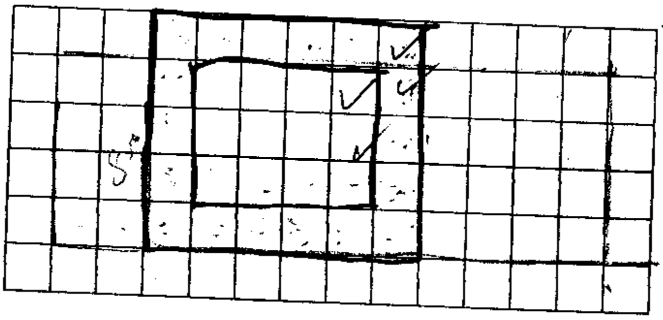
1. Find the area of the sandbox and the area of the rubber matting.

Sandbox area: 24 square feet Rubber matting area: 56 square feet

More children are using the playground, so the committee decides to double the area of the sandbox.

2. Design a new rectangular sandbox that has double the area of the original sandbox. On the grid below, make a scale drawing of the new sandbox and the surrounding rubber matting.

SCALE: □ = 4 square feet
2 feet



3. How many square feet of rubber matting will they need? 36 square feet

4. What is the length and width of the new sandbox? length 8 feet width 6 feet

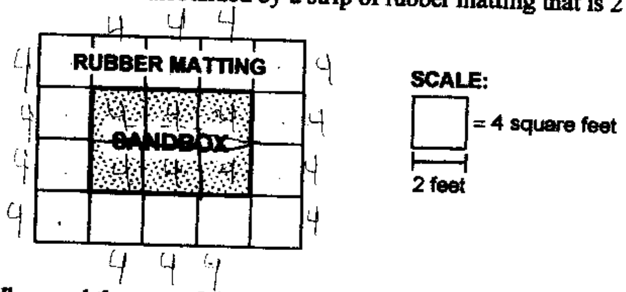
Student C counts squares and multiplies by the scale factor of 4 to find the area of the sandbox and rubber matting in part 1. The student shows doubling the area to find the area of 48 sq. ft. for the new sandbox. However, the student cannot work backward from the area with the scale factor included to the representation of the area on the graph. Instead of dividing the 48 sq. ft. by 4 to find the number of squares, Student C draws a sandbox of with 48 squares, which would represent 192 sq. ft. The student is able to use the linear scale of 2 to find the dimensions of the sandbox in his design.

Student C

The playground committee decides to make a sandbox area for toddlers.
For safety reasons, the sandbox must be surrounded by a strip of rubber matting that is 2 feet wide.

Here is a scale drawing of the sandbox.

4x6=24



1. Find the area of the sandbox and the area of the rubber matting.

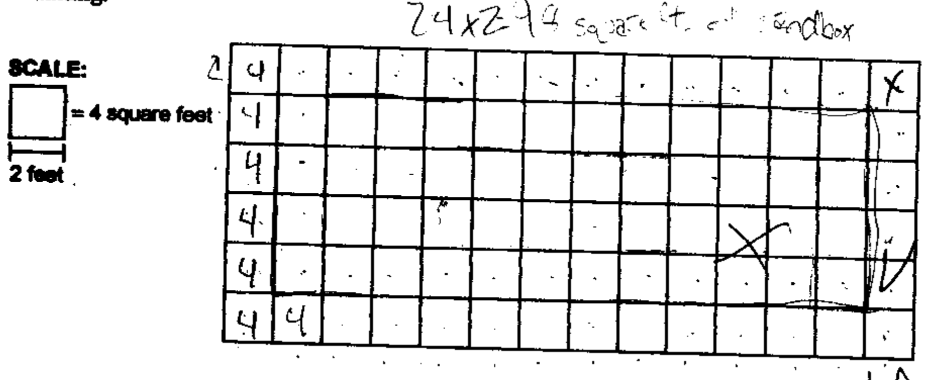
4 x 14 = 56

Sandbox area: 24 ✓ square feet **Rubber matting area:** 56 ✓ square feet

More children are using the playground, so the committee decides to double the area of the sandbox.

2. Design a new rectangular sandbox that has double the area of the original sandbox. On the grid below, make a scale drawing of the new sandbox and the surrounding rubber matting.

24x2 48 square ft. of sandbox



3. How many square feet of rubber matting will they need? 144 ✓ square feet 1 Ft

4. What is the length and width of the new sandbox? length 24 ✓ feet 1 Ft

width 8 ✓ feet 1 F

Student D is also able to count squares and multiply by the area scale factor of 4 in part 1 and 3. Student D is also able to design a new sandbox with double the area of the sandbox in part 1. However the student is unable to use the linear scale to translate from the drawing of the sandbox to the actual dimensions represented by the drawing.

Student D

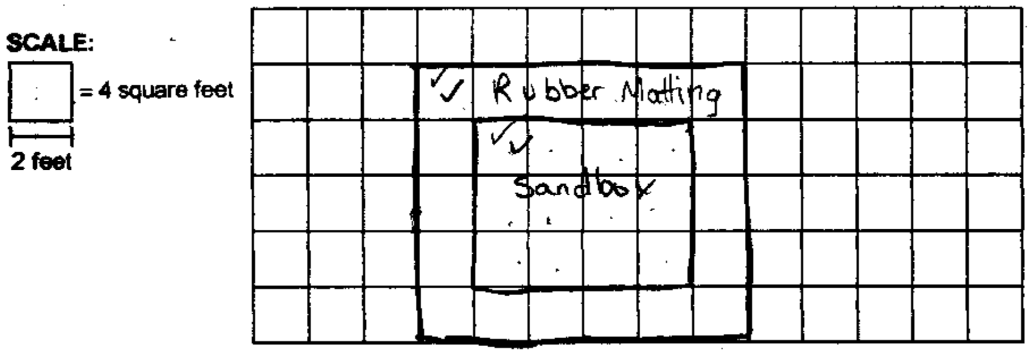
1. **Find the area of the sandbox and the area of the rubber matting.**

   Sandbox area: ___24___ square feet **Rubber matting area:** ___56___ squa

   6 × 4 = 24 14 × 4 = 56

More children are using the playground, so the committee decides to double the area of the san

2. **Design a new rectangular sandbox that has double the area of the original sandbox. On the grid below, make a scale drawing of the new sandbox and the surrounding rubber matting.**



3. **How many square feet of rubber matting will they need?** ___72___ square

4. **What is the length and width of the new sandbox?** length ___3___ feet

   width ___4___ feet

3. 12 + 6 = 18 18 × 4 = 72

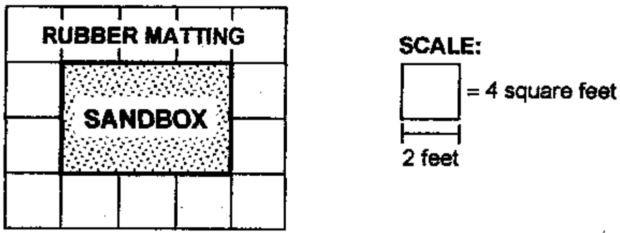
12 units

Student E is not able to find the area of the sandbox in part 1. The student multiplies the area by the linear scale factor instead of the area scale factor. The student is still able to design a new sandbox with double the area, because the number of squares doubles. In the choice of design the student has made a rectangle, which is equal to original sandboxes side by side. Student E uses the linear scale factor to find the dimensions of the new sandbox. . The student misses the constraint of the rubber matting being a strip 2 feet wide. However the student does correctly find the area for the rubber matting in his drawing.

Student E

The playground committee decides to make a sandbox area for toddlers.
For safety reasons, the sandbox must be surrounded by a strip of rubber matting that is 2 feet wide

Here is a scale drawing of the sandbox.

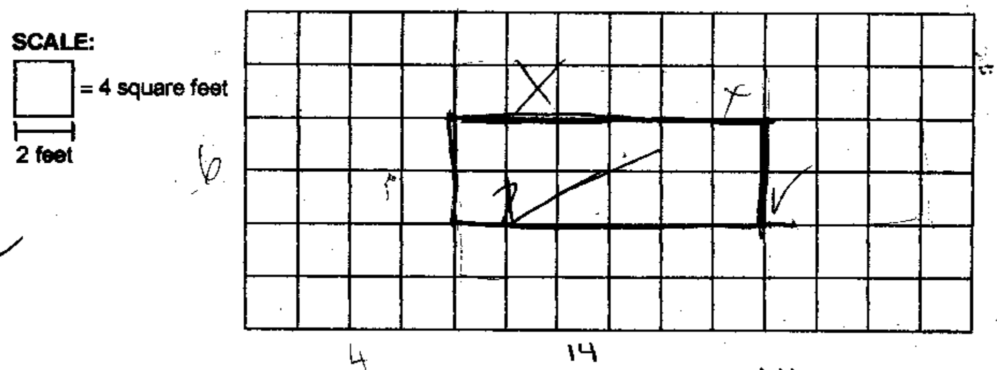


1. **Find the area of the sandbox and the area of the rubber matting.**

   Sandbox area: 12 **square feet** **Rubber matting area:** 56 **square fe**

More children are using the playground, so the committee decides to double the area of the sandbo

2. **Design a new rectangular sandbox that has double the area of the original sandbox. On the grid below, make a scale drawing of the new sandbox and the surrounding rubber matting.**



3. **How many square feet of rubber matting will they need?** [redacted] 288 **square feet**
4. **What is the length and width of the new sandbox?** **length** 4 **feet**

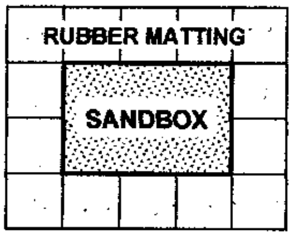
   width 12 **feet**

Student F makes the most common error of counting squares to find the area and ignoring the scale given in the task. The student is able to double the area, because the scale does not effect the number of squares in the representation. The student also ignores the scale for both linear and area measures in part 3 and 4 of the task.

Student F

The playground committee decides to make a sandbox area for toddlers.
For safety reasons, the sandbox must be surrounded by a strip of rubber matting that is 2 feet wide.

Here is a scale drawing of the sandbox.



SCALE:
□ = 4 square feet
2 feet

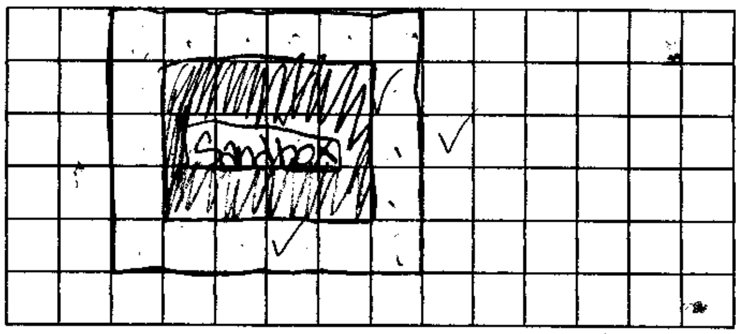
1. Find the area of the sandbox and the area of the rubber matting.

   Sandbox area: 6 square feet   Rubber matting area: 14 square feet

More children are using the playground, so the committee decides to double the area of the sandbox.

2. Design a new rectangular sandbox that has double the area of the original sandbox.
   On the grid below, make a scale drawing of the new sandbox and the surrounding rubber matting.

SCALE:
□ = 4 square feet
2 feet



3. How many square feet of rubber matting will they need? 18 square feet

4. What is the length and width of the new sandbox?   length 3 feet

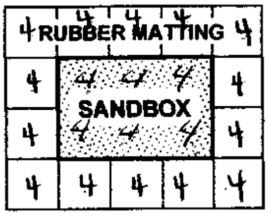
   width 4 feet

Student G is able to count squares and multiply by the scale factor of 4 to find the area for the sandbox and matting in part 1. The student may have subtracted the sandbox area from the total area to find the matting. In part 2 the student designs a sandbox that is not the obvious double of anything from part 1. The student also ignores the idea that the matting is always 2 feet wide. When calculating the area of the matting, the student leaves out one set of 4 squares along the side of the sandbox, making the total off by 16 square units. The student multiplies the dimensions by the area scale factor instead of the linear scale factor.

Student G

The playground committee decides to make a sandbox area for toddlers.
For safety reasons, the sandbox must be surrounded by a strip of rubber matting that is 2 feet wide.

Here is a scale drawing of the sandbox.



SCALE: [4] = 4 square feet
2 feet

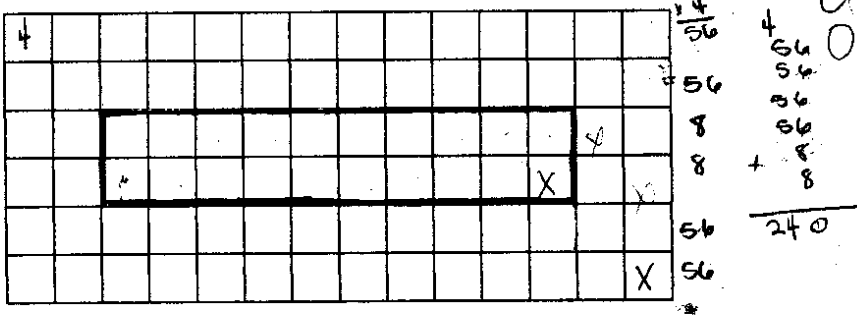
1. **Find the area of the sandbox and the area of the rubber matting.**

   Sandbox area: 24 sq square feet **Rubber matting area:** 56 sq square feet

More children are using the playground, so the committee decides to double the area of the sandbox.

2. **Design a new rectangular sandbox that has double the area of the original sandbox. On the grid below, make a scale drawing of the new sandbox and the surrounding rubber matting.**

SCALE: [4] = 4 square feet
2 feet



3. **How many square feet of rubber matting will they need?** 240 sq square feet

4. **What is the length and width of the new sandbox?** length 40 feet

   width 8 feet

Student H miscounts the squares in the sandbox or multiplies the linear scale by the area scale to find the area of the sandbox in part 1. The student gives the width of the matting instead of the area. The student designs the sandbox by looking at the pattern of filling in the whole grid except for a surrounding frame of 1 square unit. The student gives the dimensions of his sandbox without regard to the linear scale factor.

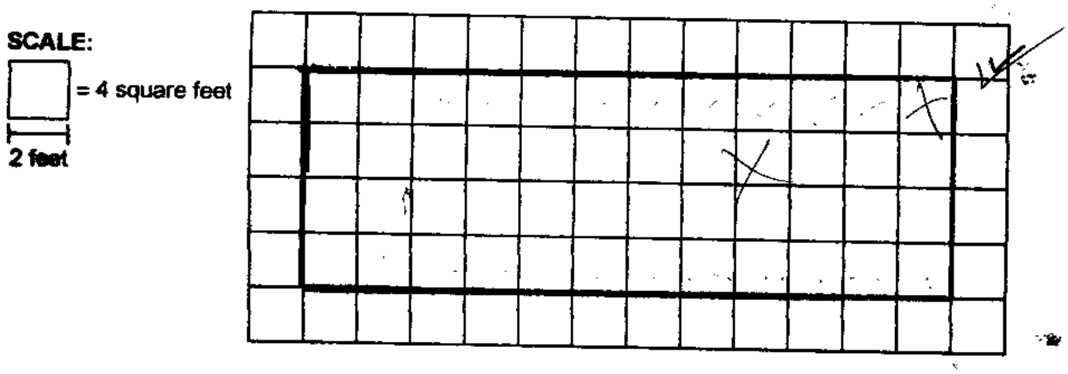
Student H

1. **Find the area of the sandbox and the area of the rubber matting.**

   Sandbox area: 8 square feet   **Rubber matting area:** 2 square

More children are using the playground, so the committee decides to double the area of the sandl

2. **Design a new rectangular sandbox that has double the area of the original sandbox. On the grid below, make a scale drawing of the new sandbox and the surrounding rubber matting.**



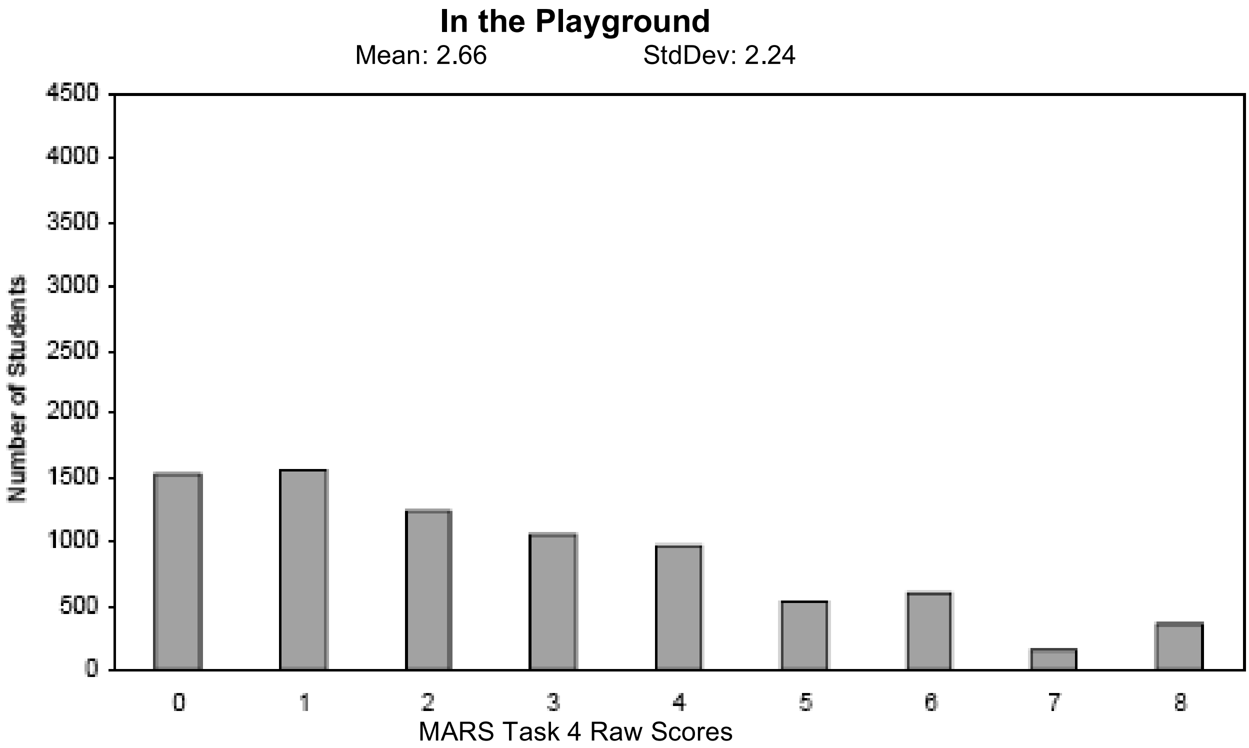
3. **How many square feet of rubber matting will they need?** 16 square fee

4. **What is the length and width of the new sandbox?** length 4 feet

   width 12 feet

Teacher Notes:

Frequency Distribution for Task 4 – Grade 6 – In the Playground



Score:	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Student Count	1535	1559	1246	1046	966	534	597	148	361
% < =	19.2%	38.7%	54.3%	67.4%	79.5%	86.2%	93.6%	95.5%	100.0%
% > =	100.0%	80.8%	61.3%	45.7%	32.6%	20.5%	13.8%	6.4%	4.5%

The maximum score available on this task is 8 points.
The minimum score for a level 3 response, meeting standards, is 4 points.

Most students, 79%, can draw a new sandbox that has one row of matting surrounding it. More than half the students, 62%, can either find the area of the original sandbox and matting or draw a sandbox with one row of matting and give one of the dimensions for the new sandbox correctly using the linear scale factor. Some students, about 33%, could find the area of the original sandbox and matting and either design a new sandbox with double the area or draw a sandbox with one row of matting and give one dimension for the new sandbox. Only 4% of the students could meet all the demands of the task including correctly applying both a linear and area scale factor to a drawing and designing a new sandbox with double the area of the original while still maintaining only one row of matting. 21% of the students scored no points on this task. 86% of those students attempted the task.

In the Playground

Points	Understandings	Misunderstandings
0	86% of the students attempted the task.	Students did not understand how to apply scale factors to a drawing. 16% of all students just counted squares in the drawing to find area for the original sandbox and matting. 8% found an area of 48 for the original sandbox, another 8% found an area of 12.
1	Students could draw a new sandbox with one row of matting along the outside.	About 14% drew no matting around the new sandbox. 32% drew matting with more than one row on at least one dimension. (Some of these overlap.)
2	Students could either find the area for the original sandbox and matting or draw a sandbox with one row of matting and use the linear scale to find one of the dimensions of the new sandbox.	Students frequently confused linear and area scale factors. They might multiply the length by 4 instead of two or multiply area by 2. In some cases students multiplied the two scale factors together before applying them to drawing.
4	Students could find the area of the parts of the original drawing. They could either design a sandbox with double the area or draw a sandbox with the proper matting and give one dimension of the new sandbox. Those students who could draw the sandbox with double the area were able to do so because they were thinking about number of squares.	9% of all students drew a rectangle equal to the entire grid minus one row for matting, thinking of a pattern rather than an area. 8% doubled the dimensions instead of the area, drawing a rectangle that was 4 x 6. 8% doubled the area of the entire play area, 80 sq. ft., making a new sandbox 4 x 10.
6	Students could find areas of the original sandbox, design a new sandbox with double the area, and find the area of the matting for the new sandbox.	Students could not apply the linear scale factor to find the dimension for their sandbox. Most of these students counted squares (3 x4 or 2 x 6).
8	Students could work with linear and area scale factors to calculate areas and dimensions of a sandbox and design a scale drawing of a sandbox with double the area.	

Based on teacher observation this is what sixth grade students knew and were able to do:

- Students could find the area of the drawing and use the appropriate scale factor to find the area of the real sandbox and matting
- Many students could draw a sandbox with one row of matting, understanding the constraint that the size of the matting does not change

Areas of difficulty for sixth graders:

- Doubling the area of the sandbox
- Using a linear scale factor to find the dimensions of the sandbox they designed
- Confusing linear and area scale factors

Questions for Reflection:

- What experiences have your students had with finding area? Do they still rely on counting squares or do they use multiplication?
- Have students had multiple experiences with doubling or tripling shapes, so that they start to see that the dimensions don't double or is this a new idea for them? How many of your students, do you think, could make a good justification for why the dimensions don't double? What kind of explanation or justification would you like them to be able to make?
- What experiences have your students had with using scale factors? Have students worked with enlarging or shrinking shapes? Do you think they understand the difference between a linear and an area scale factor?

Look at student work in part one of the task. How many of your students thought the area was:

24 sq. ft.	6 sq. ft.	12 sq. ft.	48 sq. ft.	8 sq. ft.	20 sq. ft.	40 sq. ft.	Other

What kind of thinking might have led to those errors? Which errors are related to understanding the sandbox? To understanding which scale factor to use? To understanding that a scale factor is needed?

Look at student work on designing a sandbox with double the area. How many of your students understood the constraint of one row of matting? Did they not draw any matting? Make the matting more than one row? Now look at the area of the new sandbox. How many drew a shape with dimensions of:

3 x 4 *or* 2 x 6	4 x 12	4 x 6	4 x 10	Other

What type of logic led to these specific errors? What were students not understanding about area, the sandbox, and using scale factors? How are these errors different?

Implications for Instruction:

Students at this grade level should be comfortable with finding areas of rectangular shapes by multiplying the dimensions or counting squares. They should have a good understanding of the difference between linear measures and area measures. A big idea for middle grade students is to develop proportional reasoning, which would include understanding scale factors. Students should work with recording dimensions of objects, rooms, playgrounds, maps, etc. by making their own accurate scale drawings. They should have frequent opportunities to translate between scale drawings and giving the dimensions and areas for the real object.

While students are working with growing patterns, they should be challenged to think about and justify why doubling the dimensions does not double the area. A lesson might be designed to predict the area of shape when dimensions are doubled or tripled and then make a model to test this out. Shapes should not be limited to rectangular objects, but should include triangles, trapezoids, parallelograms, and even compound shapes like an L-shaped room. Students should also work with understanding or visualizing the distortion of changing only one dimension or changing both dimensions using different scale factors. Students' experiences should not be limited to enlarging shapes, but should also include working with shrinking shapes.

These types of activities help students understand the two-dimensionality of area and build an understanding of scale. They also lay the foundation for understanding similarity at later grades.

Teacher Notes: