## Breakfast Time

This problem gives you the chance to:

- calculate costs and charges for a group

1. Linda had breakfast in a café.

It cost $\$ 12.40$. She paid with a $\$ 20$ bill.
How much change did Linda get? \$ $\qquad$
Show how you figured it out.

2.


A group of nine people had the basic continental breakfast.
How much did they pay in all?
\$ $\qquad$
Show your work.
3. A different group of people had the basic continental breakfast. They paid $\$ 32$ in all.
How many people were in the group? $\qquad$ Show how you figured it out.

| Breakfast Time | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - calculate costs and charges for a group <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Gives correct answer: $\$ 7.60$ <br> Shows work such as: $20.00-12.40$ | $\begin{aligned} & 1 \\ & 1 \end{aligned}$ | 2 |
| 2. Gives correct answer: $\mathbf{\$ 5 7 . 6 0}$ <br> Shows work such as: 6.40 x 9 | $\begin{aligned} & 1 \\ & 1 \end{aligned}$ | 2 |
| 3. Gives correct answer: 5 <br> Shows work such as: $32 \div 6.40$ or repeated subtraction | 1 | 2 |
| Total Points |  | 6 |

## Breakfast Time

Work the task. Look at the rubric. What are the important mathematical ideas in this task?

Look at student work in part 1. How many of your students:

- Chose subtraction and could do the calculations correctly: $\qquad$
- Chose subtraction, set up the problem correctly, but made a regrouping error \$8.40: $\qquad$
- Tried to subtract $\$ 20$ from $\$ 12.40$ : $\qquad$
- Used multiplication $\qquad$
- Other $\qquad$
Look at work in part 2.
- How many students used multiplication?
- How many students used repeated addition?
- How many students chose a different operation?
- How many students used extra numbers in their calculations?

Did students seem to be able to use place value appropriately in this part?
Finally look at part 3, finding the number of people if the tab was $\$ 32$.

- How many students correctly used division?
- How many students correctly used multiplication (or guess and check)?
- How many students had an incorrect answer of 4? $\qquad$
- How many students had an incorrect answer of 20? $\qquad$
What might these students been thinking? What are the misconceptions they have about this situation?
What concerns you about their understanding of place value and computations with decimals?
What concerns you about students' ability to chose the correct operation?
What structures do you have in classroom to help students who are struggling? What structures are in place at your school to help students who are struggling before they fall further behind and need intervention in middle school?


## Looking at Student Work on Breakfast Time

Student A is able to choose the proper operations and work efficiently with decimals, including division with decimals. Notice how the student uses labels to make sense of the operations and what is being calculated.

## Student A

1. Linda had breakfast in a café. It cost $\$ 12.40$. She paid with a $\$ 20$ bill. How much change did Linda get? \$ Show how you figured it out.

2. 



A group of nine people had the basic continental breakfast. How much did they pay in all?

$$
\begin{aligned}
& \$ 6.40 \text { (\$ per meal) } \\
& \times 99 \text { (\# of meals } \\
& \$ 57,60
\end{aligned}
$$

3. A different group of people had the basic continental breakfast. They paid $\$ 32$ in all.
How many people were in the group?
Show how you figured it out.


Student B uses a guess and check strategy, but is thoughtful about choosing the guess. How do we help students become more effective in their number choices? How do we help them apply things they already know when using this strategy?

## Student B

1. Linda had breakfast in a café.

It cost $\$ 12.40$. She paid with a $\$ 20$ bill.
How much change did Linda get? $\$ 7.60 \mathrm{~V}$
Show how you figured it out.

2.


A group of nine people had the basic continental breakfast.
How much did they pay in all?
Show your work.

3. A different group of people had the basic continental breakfast.

They paid $\$ 32$ in all.
How many people were in the group?
 v Show how you figured it out.


$$
\begin{aligned}
& \text { I guess a number that equal } \$ 32 \text {. } \\
& \text { 26.40 BecanseI guess } \\
& \frac{155 k}{532.00} \mathrm{~N} \\
& \begin{array}{l}
\text { a number that } \\
\text { very }
\end{array} \\
& \begin{array}{l}
\text { very close to } \\
\$ 32 \text { because } \\
\text { Ibid } 6 \times 5=30
\end{array}
\end{aligned}
$$



Student C is able to work part 2 and 3 of the task correctly. The student solves part 3 by using guess and check and needs 3 guesses before arriving at the solution. The student is confused about subtraction with decimals. What experiences would help this student?

## Student C

- calculate costs and charges tor a group

1. Linda had breakfast in a café.

It cost $\$ 12.40$. She paid with a $\$ 20$ bill. How much change did Linda get? \$ Show how you figured it out.

2.

Student D doesn't understand some basics about dividing by decimals. The student seems to have an internal misconception that when dividing, "the shorter number" goes on the outside. The student was successful on part 1 and 2 of the task. What contextual clues should have alerted the student that 20 people was an unreasonable answer?

## Student D

3. A different group of people had the basic continental breakfast. They paid $\$ 32$ in all. How many people were in the group?


Show how you figured it out.


Student E solves part 1 and 2 of the task, but struggles with the meaning of part 3. The student inserts a new number into the problem to make the division easier.

## Student E

3. A different group of people had the basic continental breakfast.

They paid $\$ 32$ in all.
How many people were in the group?


Show how you figured it out.

$$
32 / 4=8 x
$$

Student F has full marks for part 1. The student tries to invent her own algorithm for multiplying decimals by decomposing the number into dollars and cents. However the student doesn't understand how place value comes into play between the two groups. Thinking of digits separately from their place value leads to many computational errors. Students need to think about the value of the digit in all their mathematical conversations. Student F chooses an incorrect operation in part 3. The student also attempts to use the 9 from part 2 above. How could the student have finished the problem successfully using the subtraction problem? What would she need to do next?

## Student $\mathbf{F}$

2. 



A group of nine people had the basic continental breakfast.
How much did they pay in all?
Show your work.

3. A different group of people had the basic continental breakfast. They paid $\$ 32$ in all.
How many people were in the group?
Show how you figured it out.


Student G has a strategy for finding the number of people who spent $\$ 32$, but doesn't know how to interpret his answer. How could this work have led to a correct solution? What does the student need to think about to interpret this information?

## Student G

3. A different group of people had the basic continental breakfast. They paid $\$ 32$ in all.
How many people were in the group?


Show how you figured it out.



Fifth Grade

Student H does not show much of her work. In part 1 the student has the answer typical of someone who doesn't regroup when subtracting. In part 2 the student multiplies the dollars by 9 but keeps the cents the same (no multiplication). In part 3 the student attempts to use estimation to solve the problem. If 1 breakfast is about $\$ 6$, the 2 breakfasts would be about $\$ 12$. While there is sense-making of the context, why doesn't the strategy work?

## Student H

1. Linda had breakfast in a café. It cost $\$ 12.40$. She paid with a $\$ 20$ bill. How much change did Linda get?
 Show how you figured it out.

2. 

| Basic Continental Breakfast |
| :---: |
| $\$ 6.40$ |
| each |

A group of nine people had the basic continental breakfast. How much did they pay in all?


Show your work.

3. A different group of people had the basic continental breakfast. They paid $\$ 32$ in all.
How many people were in the group?
Show how you figured it out.

$9^{\text {people }}=54^{\circ}$
7 people $=32 \$$



Student I subtracts incorrectly in part 1. The student doesn't know to add zeroes to the end of the $\$ 20$. In part 2 the student tries to multiply, treating the dollars and cents as separate units. The student did understand how to add the cents back to the dollars, but only added 6 groups of 40 cents instead of 9 groups. In the process of compartmentalizing the numbers, the student lost some of the facts of the problem. The student had an idea about division in part 3 but clearly didn't understand how to do the computation.

## Student I

1. Linda had breakfast in a café. It cost $\$ 12.40$. She paid with $\$ \$ 20$ bill. How much change did Linda get? Show how you figured it out.


2. 



A group of nine people had the basic continental breakfast.

How much did they pay in all?
Show your work.


3. A different group of people had the basic continental breakfast. They paid $\$ 32$ in all. How many people were in the group? Show how you figured it out.


Student J chooses the incorrect operation in part 2 and 3. Notice that the student changes the $\$ 32$ to $\$ 3.20$ to make it the right size for the operation. What activities help students learn to choose the correct operations?

## Student J

1. Linda had breakfast in a café.

It cost $\$ 12.40$. She paid with a $\$ 20$ bill.
How much change did Linda get?
Show how you figured it out.

2.


A group of nine people had the basic continental breakfast.
How much did they pay in all?
Show your work.

3. A different group of people had the basic continental breakfast. They paid $\$ 32$ in all.
How many people were in the group?
Show how you figured it out.


| Student Task | Calculate costs and charges using decimals for a group buying breakfast. <br> Core Idea 2 <br> Number and <br> Operation |
| :--- | :--- |
| Understand the meanings of operations and how they relate to each <br> other, make reasonable estimates, and compute fluently. <br> - <br> Develop and use strategies to solve problems involving number <br> operations with fractions and decimals relevant to students' <br> experience. |  |
| -Reason about and solve problem situations that involve more <br> than one operation in multi-step problems. |  |
| Core Idea 1 <br> Number <br> Properties | Understand the place-value structure of the base-ten number <br> system including being able to represent and compare rational <br> numbers. |

The mathematics of this task:

- Choosing operations
- Understanding decimal place value
- Calculating with decimals

Based on teacher observations, this is what fifth graders know and are able to do:

- Use correct money notation
- Choose appropriate operation
- Use multiplication to help solve a division problem
- Multiply decimals
- Understand the concept of change

Areas of difficulty for fifth graders:

- Division with decimals
- Subtraction with regrouping or adding in the extra zeroes before subtracting from the $\$ 20$
- Where to put the numbers in a division problem (some students want the "shorter number" on the outside)

Task 2 - Breakfast Time
Mean: $4.82 \quad$ StdDev: 1.65

Table 26: Frequency Distribution of MARS Test Task 2, Grade 5

| Task 2 <br> Scores | Student <br> Count | \% at or <br> below | \% at or <br> above |
| :---: | :---: | :---: | :---: |
| 0 | 296 | $3.4 \%$ | $100.0 \%$ |
| 1 | 276 | $6.5 \%$ | $96.6 \%$ |
| 2 | 502 | $12.2 \%$ | $93.5 \%$ |
| 3 | 523 | $18.2 \%$ | $87.8 \%$ |
| 4 | 1097 | $30.6 \%$ | $81.8 \%$ |
| 5 | 1416 | $46.7 \%$ | $69.4 \%$ |
| 6 | 4682 | $100.0 \%$ | $53.3 \%$ |

Figure 35: Bar Graph of MARS Test Task 2 Raw Scores, Grade 5


The maximum score available for this task is 6 points.
The minimum score needed for a level 3 response, meeting standards, is 4 points.
Most students, about $96 \%$, could set up the subtraction of decimals to find change in part 1. Most students, $82 \%$, could also multiply with decimals to find the cost of breakfast for 5 people. More than half the students could meet all the demands of the task including using division or multiplication to find the number of people who spent $\$ 32$. Less than $4 \%$ of the students scored no points on this task. All the students in the sample with this score attempted the task.

## Breakfast Time

| Points | Understandings | Misunderstandings |
| :---: | :--- | :--- |
| $\mathbf{0}$ | All the students in the sample <br> with this score attempted the <br> task. | Students did not know how to subtract with <br> decimals. 4\% of the students didn't regroup <br> (\$8.40) |
| $\mathbf{2}$ | Students could subtract decimals <br> to calculate change. | Students had difficulty with multiplying <br> decimals. Some students decomposed the <br> number into dollars and cents and didn't <br> know how to put them back together. Other <br> students made arithmetic errors. |
| $\mathbf{4}$ | Students could subtract and <br> multiply with decimals. | Students did not know how to divide with <br> decimals. Many students were able to use <br> multiplication and guess and check to solve <br> the problem. 7\% of the students reversed <br> the order of the numbers for division and <br> had an answer of 20 people. 7\% had an <br> answer of 4 people. |
| $\mathbf{5}$ | About half the students with this score <br> made a calculation error in part 1. The <br> other half made a calculation error in the <br> division in part 3. |  |
| $\mathbf{6}$ | Students could reason with <br> decimals in context to solve <br> subtraction, multiplication, and <br> division problems. |  |

## Implications for Instruction

Students need more practice with problems in context. They need to be able to recognize the operation needed to solve the problem. Students, who are still struggling with operation, might benefit from learning how to use bar models to mirror the action of word problems and thus help them choose the correct operation.
Although students have been working with money notation since early grades, some students are still struggling with the idea of place value. There is evidence of students thinking about digits in isolation with no place value attached. "I multiplied 6 and 4 and got 24 ." The new level of understanding for this grade level is to start understanding decimals as representing fractional parts of a whole, not as a convenient way of segregating numbers or units. One effective way to help students learn academic vocabulary is have the teacher try to weave in definitions when using the mathematical vocabulary in communicating with students and during instruction. Students might benefit from some number talks with decimals, so that the issue of place value can be brought up with clarifying questions.
Students should be comfortable using decimals in money for addition, subtraction, multiplication, and division. Often students have a logic for their misconceptions based on a faulty generalization from earlier work. There is evidence that some students think that the $\$ 32$ needs to go on the outside ( 32 into 6.40 ) because the 32 is shorter than 6.40. To let go of these misconceptions, students need to confront them explicitly and see why they don't make sense.

## Ideas for Action Research- Exploring Place Value

Students need opportunities to play with numbers and explore how they operate. One idea is to do an investigation around exact sums and differences. (from Teaching StudentCentered Mathematics by John Van de Walle). Give students a sum such as $83.53+7.4+$ 0.649 . First ask students to estimate the answer and explain how they made the estimate. Second, have students get an exact answer (without using a calculator) and explain how they got their answer. Finally ask students to generalize about how to add numbers with decimals with any two numbers.

When student pairs have finished all three parts, have them share their strategies with other students and test their strategies with other numbers. This same format can be used with subtracting decimals.

A second way to get students to explore numbers is to do a matching activity. Problems in the matching set are designed to bring up common errors. As students work through they activity they need to explain how and why they matched the different representations. During the activity there is nothing to discuss except the mathematics of place value.
Matching activities can be set up for looking at place value or computations with decimals. Here is one example from the new web magazine, Educational Design: http://www.educationaldesigner.org/ed/volume1/issue1

## Appendices: Two further examples of multiple representation tasks

## Example 1: Visualising and ordering decimals and fractions (DfES, 2005)

In this example, students are first asked to order a set of decimals into order of size. This exposes many misconceptions (e.g., size is related to the number of digits). They are then given area and/or number line diagrams to help interpret the numbers. Finally groups of cards are arranged in order of size. The whole process is repeated with the fractions and, over several sessions the six sets are combined.


A final choice of activities around place value is to give students 3 or 4 problems for any operation you want to explore with decimals. Plan the problem so that typical errors will occur. Then give students number lines to check their answers.

