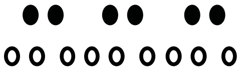
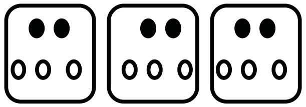
Ratios and Proportional Relationships 6.RP

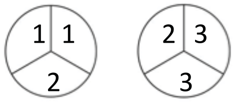
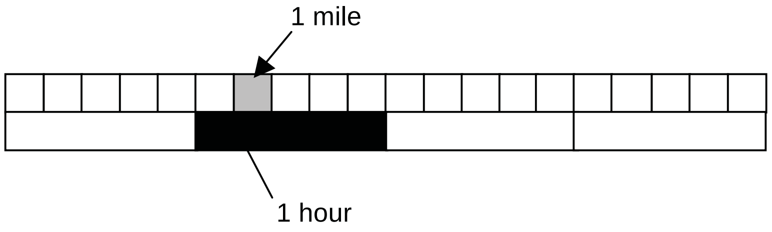
Common Core Cluster

Understand ratio concepts and use ratio reasoning to solve problems.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **ratio, equivalent ratios, tape diagram, unit rate, part-to-part, part-to-whole, percent**

A detailed progression of the Ratios and Proportional Relationships domain with examples can be found at http://commoncoretools.wordpress.com/

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*	**6.RP.1** A ratio is the comparison of two quantities or measures. The comparison can be part-to-whole (ratio of guppies to all fish in an aquarium) or part-to-part (ratio of guppies to goldfish). Example 1: A comparison of 6 guppies and 9 goldfish could be expressed in any of the following forms: $\frac{6}{9}$, 6 to 9 *or* 6:9. If the number of guppies is represented by black circles and the number of goldfish is represented by white circles, this ratio could be modeled as  These values can be regrouped into 2 black circles (goldfish) to 3 white circles (guppies), which would reduce the ratio to, $\frac{2}{3}$, 2 to 3 or 2:3.  Students should be able to identify and describe any ratio using "For every _____ ,there are _____" In the example above, the ratio could be expressed saying, "For every 2 goldfish, there are 3 guppies".
6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with $b \neq 0$, and use rate language in the	**6.RP.2** A unit rate expresses a ratio as part-to-one, comparing a quantity in terms of one unit of another quantity. Common unit rates are cost per item or distance per time.

context of a ratio relationship. *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”*[1] [1] Expectations for unit rates in this grade are limited to non-complex fractions.	Students are able to name the amount of either quantity in terms of the other quantity. Students will begin to notice that related unit rates (i.e. miles / hour and hours / mile) are reciprocals as in the second example below. At this level, students should use reasoning to find these unit rates instead of an algorithm or rule. In 6th grade, students are not expected to work with unit rates expressed as complex fractions. Both the numerator and denominator of the original ratio will be whole numbers. <u>Example 1:</u> There are 2 cookies for 3 students. What is the amount of cookie each student would receive? (i.e. the unit rate) *Solution:* This can be modeled as shown below to show that there is $\frac{2}{3}$ of a cookie for 1 student, so the unit rate is $\frac{2}{3}$: 1.  <u>Example 2:</u> On a bicycle Jack can travel 20 miles in 4 hours. What are the unit rates in this situation, (the distance Jack can travel in 1 hour and the amount of time required to travel 1 mile)? *Solution:* Jack can travel 5 miles in 1 hour written as $\frac{5\,mi}{1\,hr}$ and it takes $\frac{1}{5}$ of a hour to travel each mile written as $\frac{\frac{1}{5}\text{hr}}{1\text{mi}}$. Students can represent the relationship between 20 miles and 4 hours. 

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

6.RP.3 Ratios and rates can be used in ratio tables and graphs to solve problems. Previously, students have used additive reasoning in tables to solve problems. To begin the shift to proportional reasoning, students need to begin using multiplicative reasoning. To aid in the development of proportional reasoning the cross-product algorithm is ***not*** expected at this level. When working with ratio tables and graphs, ***whole number*** measurements are the expectation for this standard.

Example 1:

At Books Unlimited, 3 paperback books cost $18. What would 7 books cost? How many books could be purchased with $54.

Solution: To find the price of 1 book, divide $18 by 3. One book costs $6. To find the price of 7 books, multiply $6 (the cost of one book times 7 to get $42. To find the number of books that can be purchased with $54, multiply $6 times 9 to get $54 and then multiply 1 book times 9 to get 9 books. Students use ratios, unit rates and multiplicative reasoning to solve problems in various contexts, including measurement, prices, and geometry. Notice in the table below, a multiplicative relationship exists between the numbers both horizontally (times 6) and vertically (ie. $1 \bullet 7 = 7$; $6 \bullet 7 = 42$). Red numbers indicate solutions.

Number of Books (n)	Cost (C)
1	6
3	18
7	42
9	54

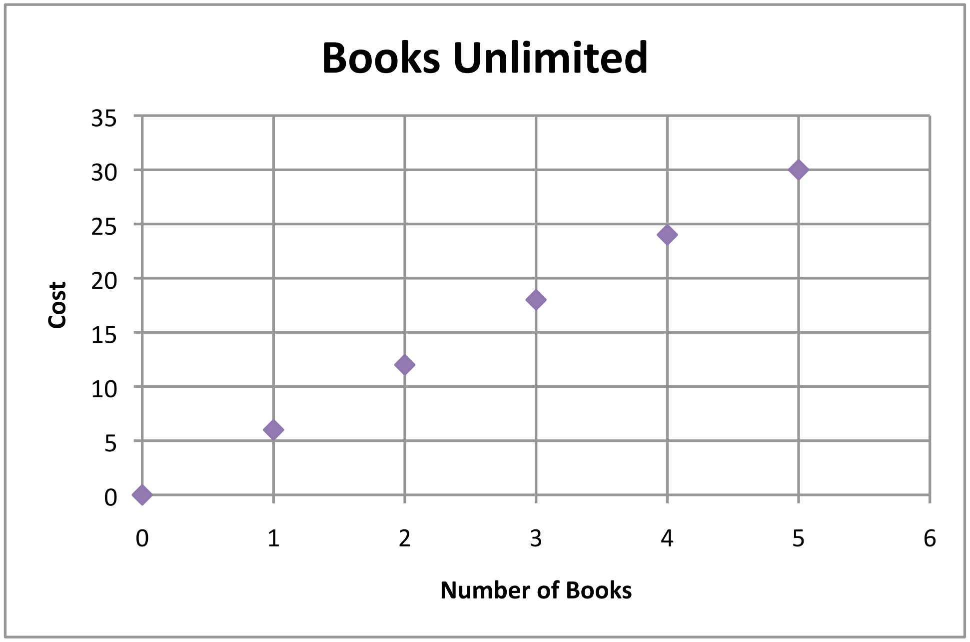
Students use tables to compare ratios. Another bookstore offers paperback books at the prices below. Which bookstore has the best buy? Explain your answer.

Number of Books (n)	Cost (C)
4	20
8	40

To help understand the multiplicative relationship between the number of books and cost, students write equations to express the cost of any number of books. Writing equations is foundational for work in 7th grade. For example, the equation for the first table would be $C = 6n$, while the equation for the second bookstore is $C = 5n$.

The numbers in the table can be expressed as ordered pairs (number of books, cost) and plotted on a coordinate plane.

Students are able to plot ratios as ordered pairs. For example, a graph of Books Unlimited would be:



Example 2:
Ratios can also be used in problem solving by thinking about the total amount for each ratio unit.
The ratio of cups of orange juice concentrate to cups of water in punch is 1: 3. If James made 32 cups of punch, how many cups of orange did he need?

Solution: Students recognize that the total ratio would produce 4 cups of punch. To get 32 cups, the ratio would need to be duplicated 8 times, resulting in 8 cups of orange juice concentrate.

Example 3:
Using the information in the table, find the number of yards in 24 feet.

Feet	3	6	9	15	24
Yards	1	2	3	5	?

Solution:

There are several strategies that students could use to determine the solution to this problem:

- Add quantities from the table to total 24 feet (9 feet and 15 feet); therefore the number of yards in 24 feet must be 8 yards (3 yards and 5 yards).
- Use multiplication to find 24 feet: 1) 3 feet x 8 = 24 feet; therefore 1 yard x 8 = 8 yards, or 2) 6 feet x 4 = 24 feet; therefore 2 yards x 4 = 8 yards.

<u>Example 4:</u>

Compare the number of black circles to white circles. If the ratio remains the same, how many black circles will there be if there are 60 white circles?

● ● ● ● ○ ○ ○

Black	4	40	20	60	?
White	3	30	15	45	60

Solution:

There are several strategies that students could use to determine the solution to this problem

- Add quantities from the table to total 60 white circles (15 + 45). Use the corresponding numbers to determine the number of black circles (20 + 60) to get 80 black circles.
- Use multiplication to find 60 white circles (one possibility 30 x 2). Use the corresponding numbers and operations to determine the number of black circles (40 x 2) to get 80 black circles.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

Students recognize the use of ratios, unit rate and multiplication in solving problems, which could allow for the use of fractions and decimals.

<u>Example 1:</u>

In trail mix, the ratio of cups of peanuts to cups of chocolate candies is 3 to 2. How many cups of chocolate candies would be needed for 9 cups of peanuts.

Peanuts	**Chocolate**
3	2

Solution:

One possible solution is for students to find the number of cups of chocolate candies for 1 cup of peanuts by dividing both sides of the table by 3, giving $\frac{2}{3}$ cup of chocolate for each cup of peanuts. To find the amount of chocolate needed for 9 cups of peanuts, students multiply the unit rate by nine (9 • $\frac{2}{3}$), giving 6 cups of chocolate.

Example 2:

If steak costs $2.25 per pound, how much does 0.8 pounds of steak cost? Explain how you determined your answer.

Solution:

The unit rate is $2.25 per pound so multiply $2.25 x 0.8 to get $1.80 per 0.8 lb of steak.

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

This is the students' first introduction to percents. Percentages are a rate per 100. Models, such as percent bars or 10 x 10 grids should be used to model percents.

Students use ratios to identify percents.

Example 1:

What percent is 12 out of 25?

Solution: One possible solution method is to set up a ratio table: Multiply 25 by 4 to get 100. Multiplying 12 by 4 will give 48, meaning that 12 out of 25 is equivalent to 48 out of 100 or 48%.

Part	**Whole**
12	25
?	100

Students use percentages to find the part when given the percent, by recognizing that the whole is being divided into 100 parts and then taking a part of them (the percent).

Example 2:

What is 40% of 30?

Solution: There are several methods to solve this problem. One possible solution using rates is to use a 10 x 10 grid to represent the whole amount (or 30). If the 30 is divided into 100 parts, the rate for one block is 0.3. Forty percent would be 40 of the blocks, or 40 x 0.3, which equals 12.

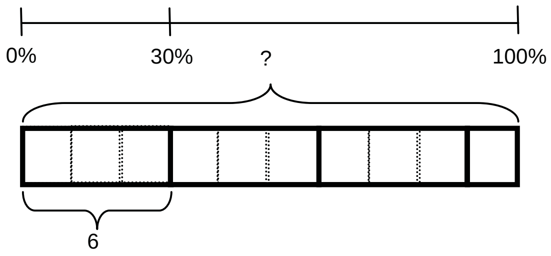
See the weblink below for more information.

http://illuminations.nctm.org/LessonDetail.aspx?id=L249

Students also determine the whole amount, given a part and the percent.

Example 3:

If 30% of the students in Mrs. Rutherford's class like chocolate ice cream, then how many students are in Mrs. Rutherford's class if 6 like chocolate ice cream?



(*Solution:* 20)

<u>Example 4:</u>
A credit card company charges 17% interest fee on any charges not paid at the end of the month. Make a ratio table to show how much the interest would be for several amounts. If the bill totals $450 for this month, how much interest would you have to be paid on the balance?
Solution:

Charges	$1	$50	$100	$200	$450
Interest	$0.17	$8.50	$17	$34	?

One possible solution is to multiply 1 by 450 to get 450 and then multiply 0.17 by 450 to get $76.50.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

A ratio can be used to compare measures of two different types, such as inches per foot, milliliters per liter and centimeters per inch. Students recognize that a conversion factor is a fraction equal to 1 since the numerator and denominator describe the same quantity. For example, $\frac{12 \text{ inches}}{1 \text{ foot}}$ is a conversion factor since the numerator and denominator equal the same amount. Since the ratio is equivalent to 1, the identity property of multiplication allows an amount to be multiplied by the ratio. Also, the value of the ratio can also be expressed as $\frac{1 \text{ foot}}{12 \text{ inches}}$ allowing for the conversion ratios to be expressed in a format so that units will "cancel".

Students use ratios as conversion factors and the identity property for multiplication to convert ratio units.

<u>Example 1:</u>
How many centimeters are in 7 feet, given that 1 inch ≈ 2.54 cm.
Solution:

$$7 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} \times \frac{2.54 \text{ cm}}{1 \text{ inch}} = 7\ \cancel{\text{feet}} \times \frac{12\ \cancel{\text{inches}}}{1\ \cancel{\text{foot}}} \times \frac{2.54 \text{ cm}}{1\ \cancel{\text{inch}}} = 7 \times 12 \times 2.54 \text{ cm} = 213.36 \text{ cm}$$

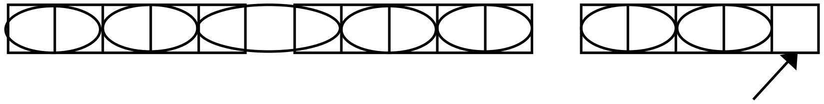
Note: Conversion factors will be given. Conversions can occur both between and across the metric and English system. Estimates are not expected.

The Number System 6.NS

Common Core Cluster

Apply and extend previous understands of multiplication and division to divide fractions by fractions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **reciprocal, multiplicative inverses, visual fraction model**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*	**6.NS.1** In 5th grade students divided whole numbers by unit fractions and divided unit fractions by whole numbers. Students continue to develop this concept by using visual models and equations to divide whole numbers by fractions and fractions by fractions to solve word problems. Students develop an understanding of the relationship between multiplication and division. Example 1: Students understand that a division problem such as $3 \div \frac{2}{5}$ is asking, "how many $\frac{2}{5}$ are in 3?" One possible visual model would begin with three whole and divide each into fifths. There are 7 groups of two-fifths in the three wholes. However, one-fifth remains. Since one-fifth is half of a two-fifths group, there is a remainder of $\frac{1}{2}$. Therefore, $3 \div \frac{2}{5} = 7\frac{1}{2}$, meaning there are $7\frac{1}{2}$ groups of two-fifths. Students interpret the solution, explaining how division by fifths can result in an answer with halves.  This section represents one-half of two-fifths Students also write contextual problems for fraction division problems. For example, the problem, $\frac{2}{3} \div \frac{1}{6}$ can be illustrated with the following word problem:

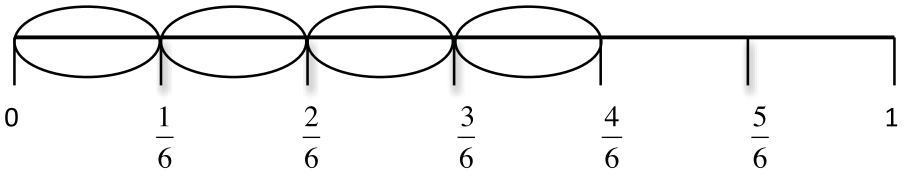
Example 2:

Susan has $\frac{2}{3}$ of an hour left to make cards. It takes her about $\frac{1}{6}$ of an hour to make each card. About how many can she make?

This problem can be modeled using a number line.

a. Start with a number line divided into thirds.

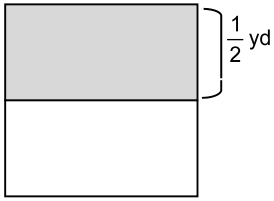
b. The problem wants to know how many sixths are in two-thirds. Divide each third in half to create sixths.

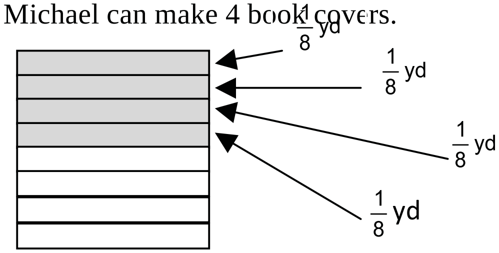


c. Each circled part represents $\frac{1}{6}$. There are four sixths in two-thirds; therefore, Susan can make 4 cards.

Example 3:

Michael has $\frac{1}{2}$ of a yard of fabric to make book covers. Each book cover is made from $\frac{1}{8}$ of a yard of fabric. How many book covers can Michael make? Solution: Michael can make 4 book covers.





Example 4:

Represent $\frac{1}{2} \div \frac{2}{3}$ in a problem context and draw a model to show your solution.

Context: A recipe requires $\frac{2}{3}$ of a cup of yogurt. Rachel has $\frac{1}{2}$ of a cup of yogurt from a snack pack. How much of the recipe can Rachel make?

Explanation of Model:

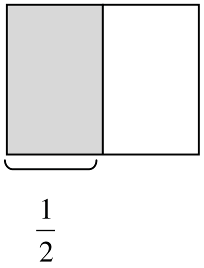
The first model shows $\frac{1}{2}$ cup. The shaded squares in all three models show the $\frac{1}{2}$ cup.

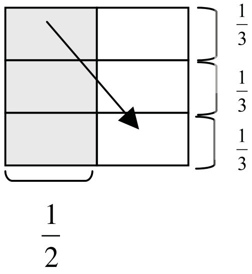
The second model shows $\frac{1}{2}$ cup and also shows $\frac{1}{3}$ cups horizontally.

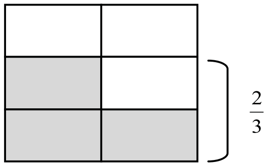
The third model shows $\frac{1}{2}$ cup moved to fit in only the area shown by $\frac{2}{3}$ of the model.

$\frac{2}{3}$ is the new referent unit (whole) .

3 out of the 4 squares in the $\frac{2}{3}$ portion are shaded. A $\frac{1}{2}$ cup is only $\frac{3}{4}$ of a $\frac{2}{3}$ cup portion, so only ¾ of the recipe can be made.







The Number System 6.NS

Common Core Cluster

Compute fluently with multi-digit numbers and find common factors and multiples.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **multi-digit**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.	**6.NS.2** In the elementary grades, students were introduced to division through concrete models and various strategies to develop an understanding of this mathematical operation (limited to 4-digit numbers divided by 2-digit numbers). In 6th grade, students become fluent in the use of the standard division algorithm, continuing to use their understanding of place value to describe what they are doing. Place value has been a major emphasis in the elementary standards. This standard is the end of this progression to address students' understanding of place value. Example 1: When dividing 32 into 8456, students should say, "there are 200 thirty-twos in 8456" as they write a 2 in the quotient. They could write 6400 beneath the 8456 rather than only writing 64.

$\begin{array}{r} 2 \\ 32\overline{)8456} \end{array}$	There are 200 thirty twos in 8456.
$\begin{array}{r} 2 \\ 32\overline{)8456} \\ \underline{6400} \\ 2056 \end{array}$	200 times 32 is 6400. 8456 minus 6400 is 2056.
$\begin{array}{r} 26 \\ 32\overline{)8456} \\ \underline{6400} \\ 2056 \end{array}$	There are 60 thirty twos in 2056.
$\begin{array}{r} 26 \\ 32\overline{)8456} \\ \underline{6400} \\ 2056 \\ \underline{1920} \\ 136 \end{array}$	60 times 32 is 1920. 2056 minus 1920 is 136.
$\begin{array}{r} 264 \\ 32\overline{)8456} \\ \underline{6400} \\ 2056 \\ \underline{1920} \\ 136 \\ \underline{128} \end{array}$	There are 4 thirty twos in 136. 4 times 32 is 128.
$\begin{array}{r} 264 \\ 32\overline{)8456} \\ \underline{6400} \\ 2056 \\ \underline{1920} \\ 136 \\ \underline{128} \\ 8 \end{array}$	The remainder is 8. There is not a full thirty two in 8; there is only part of a thirty two in 8. This can also be written as $\frac{8}{32}$ or $\frac{1}{4}$. There is ¼ of a thirty two in 8. 8456 = 264 * 32 + 8

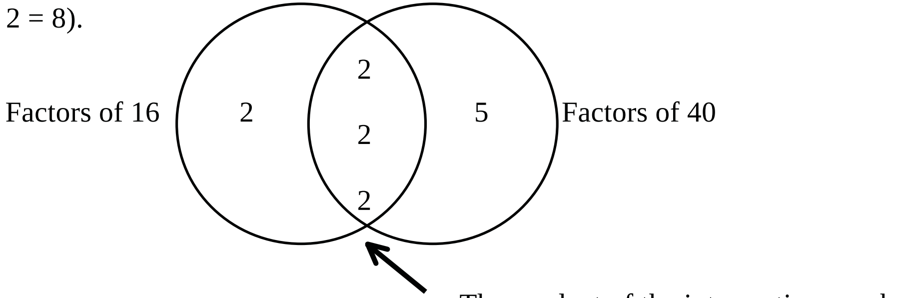
6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	**6.NS.3** Procedural fluency is defined by the Common Core as “skill in carrying out procedures flexibly, accurately, efficiently and appropriately”. In 4th and 5th grades, students added and subtracted decimals. Multiplication and division of decimals were introduced in 5th grade (decimals to the hundredth place). At the elementary level, these operations were based on concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. In 6th grade, students become fluent in the use of the standard algorithms of each of these operations. The use of estimation strategies supports student understanding of decimal operations. <u>Example 1:</u> First estimate the sum of 12.3 and 9.75. *Solution:* An estimate of the sum would be 12 + 10 or 22. Student could also state if their estimate is high or low. Answers of 230.5 or 2.305 indicate that students are not considering place value when adding.

The Number System 6.NS

Common Core Cluster

Compute fluently with multi-digit numbers and find common factors and multiples.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **greatest common factor, least common multiple, prime numbers, composite numbers, relatively prime, factors, multiples, distributive property, prime factorization**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).	In elementary school, students identified primes, composites and factor pairs (4.OA.4). In 6th grade students will find the greatest common factor of two whole numbers less than or equal to 100. For example, the greatest common factor of 40 and 16 can be found by 1) listing the factors of 40 (1, 2, 4, 5, 8, 10, 20, 40) and 16 (1, 2, 4, 8, 16), then taking the greatest common factor (8). Eight (8) is also the largest number such that the other factors are **relatively prime** (two numbers with no common factors other than one). For example, 8 would be multiplied by 5 to get 40; 8 would be multiplied by 2 to get 16. Since the 5 and 2 are relatively prime, then 8 is the greatest common factor. If students think 4 is the greatest, then show that 4 would be multiplied by 10 to get 40, while 16 would be 4 times 4. Since the 10 and 4 are not relatively prime (have 2 in common), the 4 cannot be the greatest common factor. 2) listing the prime factors of 40 (2 • 2 • 2 • 5) and 16 (2 • 2 • 2 • 2) and then multiplying the common factors (2 • 2 • 2 = 8).  The product of the intersecting numbers is the GCF Students also understand that the greatest common factor of two prime numbers is 1. Example 1: What is the greatest common factor (GCF) of 18 and 24? *Solution:* $2 * 3^2 = 18$ and $2^3 * 3 = 24$. Students should be able to explain that both 18 and 24 will have at least one factor of 2 and at least one factor of 3 in common, making $2 * 3$ or 6 the GCF.

Given various pairs of addends using whole numbers from 1-100, students should be able to identify if the two numbers have a common factor. If they do, they identify the common factor and use the distributive property to rewrite the expression. They prove that they are correct by simplifying both expressions.

Example 2:

Use the greatest common factor and the distributive property to find the sum of 36 and 8.

36 + 8 = 4 (9) + 4(2)
44 = 4 (9 + 2)
44 = 4 (11)
44 = 44✓

Example 3:

Ms. Spain and Mr. France have donated a total of 90 hot dogs and 72 bags of chips for the class picnic. Each student will receive the same amount of refreshments. All refreshments must be used.

a. What is the greatest number of students that can attend the picnic?
b. How many bags of chips will each student receive?
c. How many hotdogs will each student receive?

Solution:

a. Eighteen (18) is the greatest number of students that can attend the picnic (GCF).
b. Each student would receive 4 bags of chips.
c. Each student would receive 5 hot dogs.

Students find the least common multiple of two whole numbers less than or equal to twelve. For example, the least common multiple of 6 and 8 can be found by

1) listing the multiplies of 6 (6, 12, 18, 24, 30, …) and 8 (8, 26, 24, 32, 40…), then taking the least in common from the list (24); or
2) using the prime factorization.
   Step 1: find the prime factors of 6 and 8.
   6 = 2 • 3
   8 = 2 • 2 • 2

Step 2: Find the common factors between 6 and 8. In this example, the common factor is 2

Step 3: Multiply the common factors and any extra factors: 2 • 2 • 2 • 3 or 24 (one of the twos is in common; the other twos and the three are the extra factors.

Example 4:

The elementary school lunch menu repeats every 20 days; the middle school lunch menu repeats every 15 days. Both schools are serving pizza today. In how may days will both schools serve pizza again?

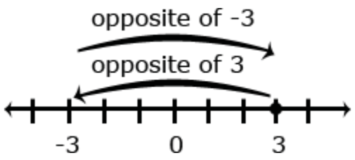
	Solution: The solution to this problem will be the least common multiple (LCM) of 15 and 20. Students should be able to explain that the least common multiple is the smallest number that is a multiple of 15 and a multiple of 20. One way to find the least common multiple is to find the prime factorization of each number: $2^2 * 5 = 20$ and $3 * 5 = 15$. To be a multiple of 20, a number must have 2 factors of 2 and one factor of 5 $(2 * 2 * 5)$. To be a multiple of 15, a number must have factors of 3 and 5. The least common multiple of 20 and 15 must have 2 factors of 2, one factor of 3 and one factor of 5 $(2 * 2 * 3 * 5)$ or 60.

The Number System 6.NS

Common Core Cluster

Apply and extend previous understandings of numbers to the system of rational numbers.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **rational numbers, opposites, absolute value, greater than, >, less than, <, greater than or equal to, ≥, less than or equal to, ≤, origin, quadrants, coordinate plane, ordered pairs, *x*-axis, *y*-axis, coordinates**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	**6.NS.5** Students use rational numbers (fractions, decimals, and integers) to represent real-world contexts and understand the meaning of 0 in each situation. Example 1: a. Use an integer to represent 25 feet below sea level b. Use an integer to represent 25 feet above sea level. c. What would 0 (zero) represent in the scenario above? *Solution:* a. -25 b. +25 c. 0 would represent sea level
6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., – (–3) = 3, and that 0 is its own opposite	**6.NS.6** In elementary school, students worked with positive fractions, decimals and whole numbers on the number line and in quadrant 1 of the coordinate plane. In 6th grade, students extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical (i.e. thermometer) which facilitates the movement from number lines to coordinate grids. Students recognize that a number and its opposite are equidistance from zero (reflections about the zero). The opposite sign (–) shifts the number to the opposite side of 0. For example, – 4 could be read as "the opposite of 4" which would be negative 4. In the example, – (–6.4) would be read as "the opposite of the opposite of 6.4" which would be 6.4. Zero is its own opposite.  Example 1: What is the opposite of $2\frac{1}{2}$? Explain your answer? *Solution:* - $2\frac{1}{2}$ because it is the same distance from 0 on the opposite side.

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	Students worked with Quadrant I in elementary school. As the *x*-axis and *y*-axis are extending to include negatives, students begin to with the Cartesian Coordinate system. Students recognize the point where the *x*-axis and *y*-axis intersect as the origin. Students identify the four quadrants and are able to identify the quadrant for an ordered pair based on the signs of the coordinates. For example, students recognize that in Quadrant II, the signs of all ordered pairs would be (–, +). Students understand the relationship between two ordered pairs differing only by signs as reflections across one or both axes. For example, in the ordered pairs (-2, 4) and (-2, -4), the *y*-coordinates differ only by signs, which represents a reflection across the *x*-axis. A change is the *x*-coordinates from (-2, 4) to (2, 4), represents a reflection across the *y*-axis. When the signs of both coordinates change, [(2, -4) changes to (-2, 4)], the ordered pair has been reflected across both axes. <u>Example1:</u> Graph the following points in the correct quadrant of the coordinate plane. If the point is reflected across the *x*-axis, what are the coordinates of the reflected points? What similarities are between coordinates of the original point and the reflected point? $\left(\frac{1}{2}, -3\frac{1}{2}\right)$ $\left(-\frac{1}{2}, -3\right)$ $(0.25, 0.75)$ *Solution:* The coordinates of the reflected points would be $\left(\frac{1}{2}, 3\frac{1}{2}\right)$ $\left(-\frac{1}{2}, 3\right)$ $(0.25, 0.75)$. Note that the *y*-coordinates are opposites. <u>Example 2:</u> Students place the following numbers would be on a number line: –4.5, 2, 3.2, $-3\frac{3}{5}$, 0.2, –2, $\frac{11}{2}$. Based on number line placement, numbers can be placed in order. *Solution:* The numbers in order from least to greatest are: –4.5, $-3\frac{3}{5}$, –2, 0.2, 2, 3.2, $\frac{11}{2}$ Students place each of these numbers on a number line to justify this order.

6.NS.7 Understand ordering and absolute value of rational numbers.

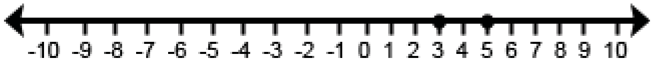
a. Interpret statements of inequality as statements about the relative position of two numbers on a number line. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*

6.NS.7 Students use inequalities to express the relationship between two rational numbers, understanding that the value of numbers is smaller moving to the left on a number line.

Common models to represent and compare integers include number line models, temperature models and the profit-loss model. On a number line model, the number is represented by an arrow drawn from zero to the location of the number on the number line; the absolute value is the length of this arrow. The number line can also be viewed as a thermometer where each point of on the number line is a specific temperature. In the profit-loss model, a positive number corresponds to profit and the negative number corresponds to a loss. Each of these models is useful for examining values but can also be used in later grades when students begin to perform operations on integers. **Operations with integers are not the expectation at this level.**

In working with number line models, students internalize the order of the numbers; larger numbers on the right (horizontal) or top (vertical) of the number line and smaller numbers to the left (horizontal) or bottom (vertical) of the number line. They use the order to correctly locate integers and other rational numbers on the number line. By placing two numbers on the same number line, they are able to write inequalities and make statements about the relationships between two numbers.

Case 1: Two positive numbers



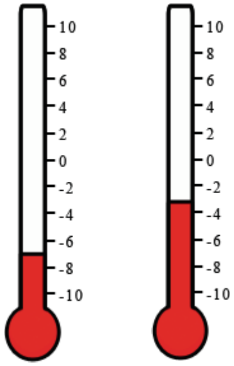
5 > 3
5 is greater than 3
3 is less than 5

Case 2: One positive and one negative number

3 > -3
positive 3 is greater than negative 3
negative 3 is less than positive 3

Case 3: Two negative numbers

-3 > -5
negative 3 is greater than negative 5
negative 5 is less than negative 3

	<u>Example 1:</u> Write a statement to compare – 4 ½ and –2. Explain your answer. *Solution:* – 4 ½ < –2 because – 4 ½ is located to the left of –2 on the number line Students recognize the distance from zero as the absolute value or magnitude of a rational number. Students need multiple experiences to understand the relationships between numbers, absolute value, and statements about order.
b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.*	Students write statements using < or > to compare rational number in context. However, explanations should reference the context rather than "less than" or "greater than". <u>Example</u> 1: The balance in Sue's checkbook was –$12.55. The balance in John's checkbook was –$10.45. Write an inequality to show the relationship between these amounts. Who owes more? *Solution:* –12.55 < –10.45, Sue owes more than John. The interpretation could also be "John owes less than Sue". <u>Example 2:</u> One of the thermometers shows -3°C and the other shows -7°C. Which thermometer shows which temperature? Which is the colder temperature? How much colder? Write an inequality to show the relationship between the temperatures and explain how the model shows this relationship.  *Solution:* • The thermometer on the left is -7; right is -3 • The left thermometer is colder by 4 degrees • Either -7 < -3 or -3 > -7 Although 6.NS.7a is limited to two numbers, this part of the standard expands the ordering of rational numbers to more than two numbers in context. <u>Example 3:</u> A meteorologist recorded temperatures in four cities around the world. List these cities in order from coldest temperature to warmest temperature: Albany 5° Anchorage -6° Buffalo -7° Juneau -9° Reno 12°

	Solution: Juneau -9° Buffalo -7° Anchorage -6° Albany 5° Reno 12°
c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*	Students understand absolute value as the distance from zero and recognize the symbols | | as representing absolute value. <u>Example 1:</u> Which numbers have an absolute value of 7 *Solution:* 7 and –7 since both numbers have a distance of 7 units from 0 on the number line. <u>Example 2:</u> What is the $\|-3\frac{1}{2}\|$? *Solution:* $3\frac{1}{2}$ In real-world contexts, the absolute value can be used to describe size or magnitude. For example, for an ocean depth of 900 feet, write | –900| = 900 to describe the distance below sea level.
d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*	When working with positive numbers, the absolute value (distance from zero) of the number and the value of the number is the same; therefore, ordering is not problematic. However, negative numbers have a distinction that students need to understand. As the negative number increases (moves to the left on a number line), the value of the number decreases. For example, –24 is less than –14 because –24 is located to the left of –14 on the number line. However, absolute value is the distance from zero. In terms of absolute value (or distance) the absolute value of –24 is greater than the absolute value of –14. For negative numbers, as the absolute value increases, the value of the negative number decreases.
6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	**6.NS.8** Students find the distance between points when ordered pairs have the same x-coordinate (vertical) or same y-coordinate (horizontal). <u>Example 1:</u> What is the distance between (–5, 2) and (–9, 2)? *Solution:* The distance would be 4 units. This would be a horizontal line since the *y*-coordinates are the same. In this scenario, both coordinates are in the same quadrant. The distance can be found by using a number line to find the distance between –5 and –9. Students could also recognize that –5 is 5 units from 0 (absolute value) and that –9 is 9 units from 0 (absolute value). Since both of these are in the same quadrant, the distance can be found by finding the difference between the distances 9 and 5. (| 9 | - | 5 |).

Coordinates could also be in two quadrants and include rational numbers.

<u>Example 2:</u>

What is the distance between $(3, -5\frac{1}{2})$ and $(3, 2\frac{1}{4})$?

Solution: The distance between $(3, -5\frac{1}{2})$ and $(3, 2\frac{1}{4})$ would be $7\frac{3}{4}$ units. This would be a vertical line since the *x*-coordinates are the same. The distance can be found by using a number line to count from $-5\frac{1}{2}$ to $2\frac{1}{4}$ or by recognizing that the distance (absolute value) from $-5\frac{1}{2}$ to 0 is $5\frac{1}{2}$ units and the distance (absolute value) from 0 to $2\frac{1}{4}$ is $2\frac{1}{4}$ units so the total distance would be $5\frac{1}{2} + 2\frac{1}{4}$ or $7\frac{3}{4}$ units.

Students graph coordinates for polygons and find missing vertices based on properties of triangles and quadrilaterals.

Expressions and Equations 6.EE

Common Core Cluster

Apply and extend previous understanding of arithmetic to algebraic expressions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **exponents, base, numerical expressions, algebraic expressions, evaluate, sum, term, product, factor, quantity, quotient, coefficient, constant, like terms, equivalent expressions, variables**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.	**6.EE.1** Students demonstrate the meaning of exponents to write and evaluate numerical expressions with whole number exponents. The base can be a whole number, positive decimal or a positive fraction (i.e. $\frac{1}{2}^{5}$ can be written $\frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2}$ which has the same value as $\frac{1}{32}$). Students recognize that an expression with a variable represents the same mathematics (ie. x^5 can be written as $x \bullet x \bullet x \bullet x \bullet x$) and write algebraic expressions from verbal expressions. Order of operations is introduced throughout elementary grades, including the use of grouping symbols, ( ), { }, and [ ] in 5th grade. Order of operations with exponents is the focus in 6th grade. Example 1: What is the value of: • 0.2^3 *Solution:* 0.008 • $5 + 2^4 \bullet 6$ *Solution:* 101 • $7^2 – 24 \div 3 + 26$ *Solution:* 67 Example 2: What is the area of a square with a side length of $3x$? *Solution:* $3x \bullet 3x = 9x^2$ Example 3: $4^x = 64$ *Solution:* $x = 3$ because $4 \bullet 4 \bullet 4 = 64$

6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 – y.	**6.EE.2** Students write expressions from verbal descriptions using letters and numbers, understanding order is important in writing subtraction and division problems. Students understand that the expression "5 times any number, *n*" could be represented with $5n$ and that a number and letter written together means to multiply. All rational numbers may be used in writing expressions when operations are not expected. Students use appropriate mathematical language to write verbal expressions from algebraic expressions. It is ***important*** for students to read algebraic expressions in a manner that reinforces that the variable represents a number. <u>Example Set 1:</u> Students read algebraic expressions: • r + 21 as "some number plus 21" as well as "r plus 21" • n • 6 as "some number times 6" as well as "n times 6" • $\frac{s}{6}$ and s ÷ 6 as "as some number divided by 6" as well as "s divided by 6" <u>Example Set 2:</u> Students write algebraic expressions: • 7 less than 3 times a number *Solution:* $3x - 7$ • 3 times the sum of a number and 5 *Solution:* $3(x + 5)$ • 7 less than the product of 2 and a number *Solution:* $2x - 7$ • Twice the difference between a number and 5 *Solution:* $2(z - 5)$ • The quotient of the sum of *x* plus 4 and 2 *Solution:* $\frac{x + 4}{2}$ Students can describe expressions such as 3 (2 + 6) as the product of two factors: 3 and (2 + 6). The quantity (2 + 6) is viewed as one factor consisting of two terms. Terms are the parts of a sum. When the term is an explicit number, it is called a constant. When the term is a product of a number and a variable, the number is called the coefficient of the variable. Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient). Variables are letters that represent numbers. There are various possibilities for the number they can represent.
b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*	

	Consider the following expression: $x^2 + 5y + 3x + 6$ The variables are x and y. There are 4 terms, x^2, 5y, 3x, and 6. There are 3 variable terms, x^2, 5y, 3x. They have coefficients of 1, 5, and 3 respectively. The coefficient of x^2 is 1, since $x^2 = 1x^2$. The term 5y represent 5y’s or $5 \bullet y$. There is one constant term, 6. The expression represents a sum of all four terms.
c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole- number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6\,s^2$ to find the volume and surface area of a cube with sides of length $s = ½$.*	Students evaluate algebraic expressions, using order of operations as needed. Problems such as example 1 below require students to understand that multiplication is understood when numbers and variables are written together and to use the order of operations to evaluate. Order of operations is introduced throughout elementary grades, including the use of grouping symbols, ( ), { }, and [ ] in 5th grade. Order of operations with exponents is the focus in 6th grade. <u>Example 1:</u> Evaluate the expression $3x + 2y$ when x is equal to 4 and y is equal to 2.4. *<u>Solution:</u>* $3 \bullet 4 + 2 \bullet 2.4$ $12 + 4.8$ 16.8 <u>Example 2:</u> Evaluate $5(n + 3) – 7n$, when $n = \frac{1}{2}$. *Solution:* $5(\frac{1}{2} + 3) – 7(\frac{1}{2})$ $5\,(3\frac{1}{2}) - 3\frac{1}{2}$ Note: $7(\frac{1}{2}) = \frac{7}{2} = 3\frac{1}{2}$ $17\frac{1}{2} - 3\frac{1}{2}$ Students may also reason that 5 groups of $3\frac{1}{2}$ take away 1 group of $3\frac{1}{2}$ would give 4 groups of $3\frac{1}{2}$. Multiply 4 times $3\frac{1}{2}$ to get 14. 14

Example 3:
Evaluate $7xy$ when $x = 2.5$ and $y = 9$

Solution: Students recognize that two or more terms written together indicates multiplication.
7 (2.5) (9)
157.5

In 5th grade students worked with the grouping symbols ( ), [ ], and { }. Students understand that the fraction bar can also serve as a grouping symbol (treats numerator operations as one group and denominator operations as another group) as well as a division symbol.

Example 4:
Evaluate the following expression when $x = 4$ and $y = 2$

$$\frac{x^2 + y^3}{3}$$

Solution:

$\frac{(4)^2 + (2)^3}{3}$ *substitute the values for x and y*

$\frac{16 + 8}{3}$ *raise the numbers to the powers*

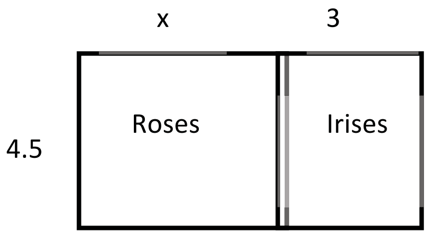
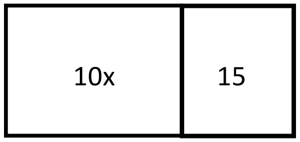
$\frac{24}{3}$ *divide 24 by 3*

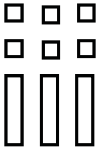
8

Given a context and the formula arising from the context, students could write an expression and then evaluate for any number.

Example 5:
It costs \$100 to rent the skating rink plus \$5 per person. Write an expression to find the cost for any number (n) of people. What is the cost for 25 people?

Solution:
The cost for any number (n) of people could be found by the expression, $100 + 5n$. To find the cost of 25 people substitute 25 in for n and solve to get $100 + 5 * 25 = 225$.

	Example 6: The expression $c + 0.07c$ can be used to find the total cost of an item with 7% sales tax, where c is the pre-tax cost of the item. Use the expression to find the total cost of an item that cost \$25. *Solution*: Substitute 25 in for c and use order of operations to simplify $c + 0.07c$ $25 + 0.07 (25)$ $25 + 1.75$ 26.75
6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression $3 (2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6 (4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.*	**6.EE.3** Students use the distributive property to write equivalent expressions. Using their understanding of area models from elementary students illustrate the distributive property with variables. Properties are introduced throughout elementary grades (3.OA.5); however, there has not been an emphasis on recognizing and naming the property. In 6th grade students are able to use the properties and identify by name as used when justifying solution methods (see example 4). Example 1: Given that the width is 4.5 units and the length can be represented by x + 2, the area of the flowers below can be expressed as $4.5(x + 3)$ or $4.5x + 13.5$.  When given an expression representing area, students need to find the factors. Example 2: The expression $10x + 15$ can represent the area of the figure below. Students find the greatest common factor (5) to represent the width and then use the distributive property to find the length $(2x + 3)$. The factors (dimensions) of this figure would be $5(2x + 3)$. 

	Example 3: Students use their understanding of multiplication to interpret *3 (2 + x) as 3 groups of (2 + x).* They use a model to represent x, and make an array to show the meaning of $3(2 + x)$. They can explain why it makes sense that $3(2 + x)$ is equal to $6 + 3x$. An array with 3 columns and $x + 2$ in each column:  Students interpret y as referring to one y. Thus, they can reason that one y plus one y plus one y ***must be*** $3y$. They also use the distributive property, the multiplicative identity property of 1, and the commutative property for multiplication to prove that $y + y + y = 3y$: Example 4: Prove that $y + y + y = 3y$ *Solution:* $y + y + y$ $y \cdot 1 + y \cdot 1 + y \cdot 1$ Multiplicative Identity $y \cdot (1 + 1 + 1)$ Distributive Property $y \cdot 3$ $3y$ Commutative Property
6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*	**6.EE.4** Students demonstrate an understanding of like terms as quantities being added or subtracted with the same variables and exponents. For example, $3x + 4x$ are like terms and can be combined as $7x$; however, $3x + 4x^2$ are not like terms since the exponents with the x are not the same. This concept can be illustrated by substituting in a value for x. For example, $9x – 3x = 6x$ not 6. Choosing a value for x, such as 2, can prove non-equivalence. $9(2) – 3(2) = 6(2)$ however $9(2) – 3(2) \overset{?}{=} 6$ $18 – 6 = 12$ $18 – 6 \overset{?}{=} 6$ $12 = 12$ $12 \neq 6$ Students can also generate equivalent expressions using the associative, commutative, and distributive properties. They can prove that the expressions are equivalent by simplifying each expression into the same form.

Example 1:

Are the expressions equivalent? Explain your answer?

$4m + 8$ $\quad 4(m+2)$ $\quad 3m + 8 + m$ $\quad 2 + 2m + m + 6 + m$

Solution:

Expression	Simplifying the Expression	Explanation
$4m + 8$	$4m + 8$	Already in simplest form
$4(m+2)$	$4(m+2)$ $4m + 8$	*Distributive property*
$3m + 8 + m$	$3m + 8 + m$ $3m + m + 8$ $4m + 8$	*Combined like terms*
$2 + 2m + m + 6 + m$	$2m + m + m + 2 + 6$ $4m + 8$	*Combined like terms* *Combined like terms*

Expressions and Equations 6.EE

Common Core Cluster

Reason about and solve one-variable equations and inequalities.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **inequalities, equations, greater than, >, less than, <, greater than or equal to, ≥, less than or equal to, ≤, profit, exceed**

<table>
<tr><th>Common Core Standard</th><th>Unpacking
What does this standard mean that a student will know and be able to do?</th></tr>
<tr><td>6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.</td><td>
<p>In elementary grades, students explored the concept of equality. In 6th grade, students explore equations as expressions being set equal to a specific value. The solution is the value of the variable that will make the equation or inequality true. Students use various processes to identify the value(s) that when substituted for the variable will make the equation true.</p>
<p><u>Example 1:</u>
Joey had 26 papers in his desk. His teacher gave him some more and now he has 100. How many papers did his teacher give him?</p>
<p>This situation can be represented by the equation $26 + n = 100$ where n is the number of papers the teacher gives to Joey. This equation can be stated as “some number was added to 26 and the result was 100.” Students ask themselves “What number was added to 26 to get 100?” to help them determine the value of the variable that makes the equation true. Students could use several different strategies to find a solution to the problem:</p>

Reasoning: 26 + 70 is 96 and 96 + 4 is 100, so the number added to 26 to get 100 is 74.
Use knowledge of fact families to write related equations:
$n + 26 = 100$, $100 - n = 26$, $100 - 26 = n$. Select the equation that helps to find n easily.
Use knowledge of inverse operations: Since subtraction “undoes” addition then subtract 26 from 100 to get the numerical value of n
Scale model: There are 26 blocks on the left side of the scale and 100 blocks on the right side of the scale. All the blocks are the same size. 74 blocks need to be added to the left side of the scale to make the scale balance.
Bar Model: Each bar represents one of the values. Students use this visual representation to demonstrate that 26 and the unknown value together make 100.

<table>
<tr><td colspan="2">100</td></tr>
<tr><td>26</td><td>n</td></tr>
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</td></tr>
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	Solution: Students recognize the value of 74 would make a true statement if substituted for the variable. $26 + n = 100$ $26 + 74 = 100$ $100 = 100$ ✓ Example 2: The equation 0.44 $s = 11$ where s represents the number of stamps in a booklet. The booklet of stamps costs 11 dollars and each stamp costs 44 cents. How many stamps are in the booklet? Explain the strategies used to determine the answer. Show that the solution is correct using substitution. *Solution:* There are 25 stamps in the booklet. I got my answer by dividing 11 by 0.44 to determine how many groups of 0.44 were in 11. By substituting 25 in for s and then multiplying, I get 11. $0.44(25) = 11$ $11 = 11$ ✓ Example 3: Twelve is less than 3 times another number can be shown by the inequality $12 < 3n$. What numbers could possibly make this a true statement? *Solution:* Since 3 • 4 is equal to 12 I know the value must be greater than 4. Any value greater than 4 will make the inequality true. Possibilities are 4.13, 6, $5\frac{3}{4}$, and 200. Given a set of values, students identify the values that make the inequality true.
6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	**6.EE.6**. Students write expressions to represent various real-world situations. Example Set 1: • Write an expression to represent Susan's age in three years, when a represents her present age. • Write an expression to represent the number of wheels, w, on any number of bicycles. • Write an expression to represent the value of any number of quarters, q. *Solutions:* • $a + 3$ • $2n$ • $0.25q$

Given a contextual situation, students define variables and write an expression to represent the situation.

Example 2:
The skating rink charges $100 to reserve the place and then $5 per person. Write an expression to represent the cost for any number of people.

n = the number of people
$100 + 5n$

No solving is expected with this standard; however, 6.EE.2c does address the evaluating of the expressions.

Students understand the inverse relationships that can exist between two variables. For example, if Sally has 3 times as many bracelets as Jane, then Jane has $\frac{1}{3}$ the amount of Sally. If S represents the number of bracelets Sally has, the $\frac{1}{3}s$ or $\frac{s}{3}$ represents the amount Jane has.

Connecting writing expressions with story problems and/or drawing pictures will give students a context for this work. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number.

Example Set 3:

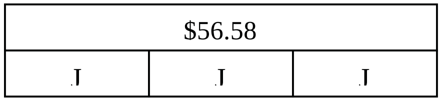
- Maria has three more than twice as many crayons as Elizabeth. Write an algebraic expression to represent the number of crayons that Maria has.
  Solution: $2c + 3$ where c represents the number of crayons that Elizabeth has

- An amusement park charges $28 to enter and $0.35 per ticket. Write an algebraic expression to represent the total amount spent.
  Solution: $28 + 0.35t$ where t represents the number of tickets purchased

- Andrew has a summer job doing yard work. He is paid $15 per hour and a $20 bonus when he completes the yard. He was paid $85 for completing one yard. Write an equation to represent the amount of money he earned.
  Solution: $15h + 20 = 85$ where h is the number of hours worked

- Describe a problem situation that can be solved using the equation $2c + 3 = 15$; where c represents the cost of an item
  Possible solution:
  Sarah spent $15 at a craft store.
    - She bought one notebook for $3.
    - She bought 2 paintbrushes for x dollars.

  If each paintbrush cost the same amount, what was the cost of one brush?

	- Bill earned $5.00 mowing the lawn on Saturday. He earned more money on Sunday. Write an expression that shows the amount of money Bill has earned. *Solution:* $5.00 + *n*
6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	**6.EE.7** Students have used algebraic expressions to generate answers given values for the variable. This understanding is now expanded to equations where the value of the variable is unknown but the outcome is known. For example, in the expression, $x + 4$, any value can be substituted for the x to generate a numerical answer; however, in the equation $x + 4 = 6$, there is only one value that can be used to get a 6. Problems should be in context when possible and use only one variable. Students write equations from real-world problems and then use inverse operations to solve one-step equations based on real world situations. Equations may include fractions and decimals with non-negative solutions. Students recognize that dividing by 6 and multiplying by $\frac{1}{6}$ produces the same result. For example, $\frac{x}{6} = 9$ and $\frac{1}{6}x = 9$ will produce the same result. Beginning experiences in solving equations require students to understand the meaning of the equation and the solution in the context of the problem. <u>Example 1:</u> Meagan spent $56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an algebraic equation that represents this situation and solve to determine how much one pair of jeans cost.  *Sample Solution:* Students might say: "I created the bar model to show the cost of the three pairs of jeans. Each bar labeled J is the same size because each pair of jeans costs the same amount of money. The bar model represents the equation $3J =$ $56.58. To solve the problem, I need to divide the total cost of 56.58 between the three pairs of jeans. I know that it will be more than $10 each because 10 x 3 is only 30 but less than $20 each because 20 x 3 is 60. If I start with $15 each, I am up to $45. I have $11.58 left. I then give each pair of jeans $3. That's $9 more dollars. I only have $2.58 left. I continue until all the money is divided. I ended up giving each pair of jeans another $0.86. Each pair of jeans costs $18.86 (15+3+0.86). I double check that the jeans cost $18.86 each because $18.86 x 3 is $56.58."

Example 2:
Julie gets paid \$20 for babysitting. He spends \$1.99 on a package of trading cards and \$6.50 on lunch. Write and solve an equation to show how much money Julie has left.

20		
1.9	6.50	money left over (m)

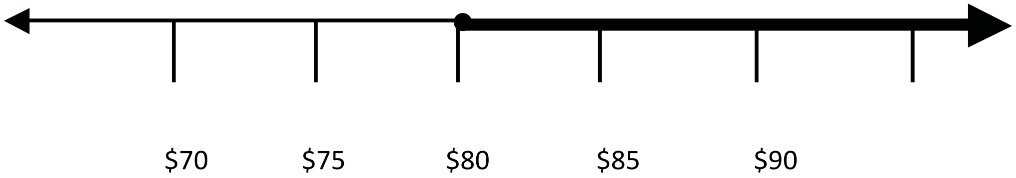
Solution: 20 = 1.99 + 6.50 + x, x = \$11.51

6.EE.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

6.EE.8 Many real-world situations are represented by inequalities. Students write inequalities to represent real world and mathematical situations. Students use the number line to represent inequalities from various contextual and mathematical situations.

Example 1:
The class must raise at least \$100 to go on the field trip. They have collected \$20. Write an inequality to represent the amount of money, *m*, the class still needs to raise. Represent this inequality on a number line.

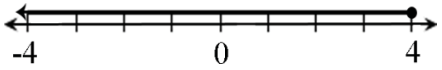
Solution:
The inequality $m \geq$ \$80 represents this situation. Students recognize that possible values can include too many decimal values to name. Therefore, the values are represented on a number line by shading.



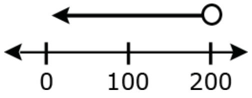
A number line diagram is drawn with an open circle when an inequality contains a < or > symbol to show solutions that are less than or greater than the number but not equal to the number. The circle is shaded, as in the example above, when the number is to be included. Students recognize that possible values can include fractions and decimals, which are represented on the number line by shading. Shading is extended through the arrow on a number line to show that an inequality has an infinite number of solutions.

Example 2:
Graph $x \leq 4$.

Solution:



Example 3:
The Flores family spent less than \$200.00 last month on groceries. Write an inequality to represent this amount and graph this inequality on a number line.

	Solution: 200 > x, where x is the amount spent on groceries. 

Expressions and Equations 6.EE

Common Core Cluster

Represent and analyze quantitative relationships between dependent and independent variables.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **dependent variables, independent variables, discrete data, continuous data**

Common Core Standard

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Unpacking

What does this standard mean that a student will know and be able to do?

6.EE.9 The purpose of this standard is for students to understand the relationship between two variables, which begins with the distinction between dependent and independent variables. The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable. Students recognize that the independent variable is graphed on the x-axis; the dependent variable is graphed on the y-axis.

Students recognize that not all data should be graphed with a line. Data that is discrete would be graphed with coordinates only. Discrete data is data that would not be represented with fractional parts such as people, tents, records, etc. For example, a graph illustrating the cost per person would be graphed with points since part of a person would not be considered. A line is drawn when both variables could be represented with fractional parts.

Students are expected to recognize and explain the impact on the dependent variable when the independent variable changes (As the x variable increases, how does the y variable change?) *Relationships should be proportional with the line passing through the origin.* Additionally, students should be able to write an equation from a word problem and understand how the coefficient of the dependent variable is related to the graph and /or a table of values.

Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective.

Example 1:

What is the relationship between the two variables? Write an expression that illustrates the relationship.

x	1	2	3	4
y	2.5	5	7.5	10

Solution:

$y = 2.5x$

Geometry 6.G

Common Core Cluster

Solve real-world and mathematical problems involving area, surface area, and volume.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **area, surface area, volume, decomposing, edges, dimensions, net, vertices, face, base, height, trapezoid, isosceles, right triangle, quadrilateral, rectangles, squares, parallelograms, trapezoids, rhombi, kites, right rectangular prism**

Common Core Standard

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Unpacking

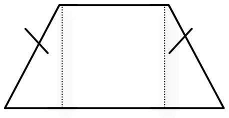
What does this standard mean that a student will know and be able to do?

6.G.1 Students continue to understand that area is the number of squares needed to cover a plane figure. Students should know the formulas for rectangles and triangles. "Knowing the formula" does not mean memorization of the formula. To "know" means to have an understanding of ***why*** the formula works and how the formula relates to the measure (area) and the figure. This understanding should be for *all* students.

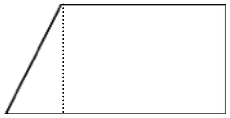
Finding the area of triangles is introduced in relationship to the area of rectangles – a rectangle can be decomposed into two congruent triangles. Therefore, the area of the triangle is ½ the area of the rectangle. The area of a rectangle can be found by multiplying base x height; therefore, the area of the triangle is ½ bh or (b x h)/2.

The following site helps students to discover the area formula of triangles.
http://illuminations.nctm.org/LessonDetail.aspx?ID=L577

Students decompose shapes into rectangles and triangles to determine the area. For example, a trapezoid can be decomposed into triangles and rectangles (see figures below). Using the trapezoid's dimensions, the area of the individual triangle(s) and rectangle can be found and then added together. Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi, and kites.



Isosceles trapezoid



Right trapezoid

Note: Students recognize the marks on the isosceles trapezoid indicating the two sides have equal measure.

Example 1:

Find the area of a right triangle with a base length of three units, a height of four units, and a hypotenuse of 5.

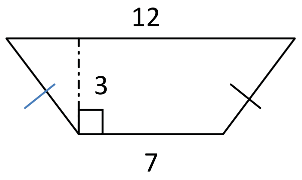
Solution:

Students understand that the hypotenuse is the longest side of a right triangle. The base and height would form the 90° angle and would be used to find the area using:

A = ½ bh
A = ½ (3 units)(4 units)
A = ½ 12 units^2
A = 6 units^2

Example 2:

Find the area of the trapezoid shown below using the formulas for rectangles and triangles.



Solution:

The trapezoid could be decomposed into a rectangle with a length of 7 units and a height of 3 units. The area of the rectangle would be 21 units^2.

The triangles on each side would have the same area. The height of the triangles is 3 units. After taking away the middle rectangle's base length, there is a total of 5 units remaining for both of the side triangles. The base length of each triangle is half of 5. The base of each triangle is 2.5 units. The area of one triangle would be ½ (2.5 units)(3 units) or 3.75 units^2.

Using this information, the area of the trapezoid would be:

21 units^2
3.75 units^2
$\underline{+3.75 \text{ units}^2}$
28.5 units^2

Example 3:

A rectangle measures 3 inches by 4 inches. If the lengths of each side double, what is the effect on the area?

Solution:

The new rectangle would have side lengths of 6 inches and 8 inches. The area of the original rectangle was 12 inches^2. The area of the new rectangle is 48 inches^2. The area increased 4 times (quadrupled).

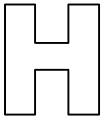
Students may also create a drawing to show this visually.

Example 4:
The lengths of the sides of a bulletin board are 4 feet by 3 feet. How many index cards measuring 4 inches by 6 inches would be needed to cover the board?

Solution:
Change the dimensions of the bulletin board to inches (4 feet = 48 inches; 3 feet = 36 inches). The area of the board would be 48 inches x 36 inches or 1728 inches^2. The area of one index card is 12 inches^2. Divide 1728 inches^2 by 24 inches^2 to get the number of index cards. 72 index cards would be needed.

Example 5:
The sixth grade class at Hernandez School is building a giant wooden H for their school. The “H” will be 10 feet tall and 10 feet wide and the thickness of the block letter will be 2.5 feet.

1. How large will the H be if measured in square feet?
2. The truck that will be used to bring the wood from the lumberyard to the school can only hold a piece of wood that is 60 inches by 60 inches. What pieces of wood (how many and which dimensions) will need to be bought to complete the project?



Solution:

1. One solution is to recognize that, if filled in, the area would be 10 feet tall and 10 feet wide or 100 ft^2. The size of one piece removed is 5 feet by 3.75 feet or 18.75 ft^2. There are two of these pieces.
   The area of the “H” would be 100 ft^2 – 18.75 ft^2 – 18.75 ft^2, which is 62.5ft^2.
   A second solution would be to decompose the “H” into two tall rectangles measuring 10 ft by 2.5 ft and one smaller rectangle measuring 2.5 ft by 5 ft. The area of each tall rectangle would be 25 ft^2 and the area of the smaller rectangle would be 12.5 ft^2. Therefore the area of the “H” would be 25 ft^2 + 25 ft^2 + 12.5 ft^2 or 62.5ft^2.
2. Sixty inches is equal to 5 feet, so the dimensions of each piece of wood are 5ft by 5ft. Cut **two pieces of wood in half to create four pieces 5 ft. by 2.5 ft.** These pieces will make the two taller rectangles. A **third piece would be cut to measure 5ft. by 2.5 ft.** to create the middle piece.

Example 6:
A border that is 2 ft wide surrounds a rectangular flowerbed 3 ft by 4 ft. What is the area of the border?

Solution:
Two sides 4 ft. by 2 ft. would be 8ft^2 x 2 or 16 ft^2
Two sides 3 ft. by 2 ft. would be 6ft^2 x 2 or 12 ft^2
Four corners measuring 2 ft. by 2 ft. would be 4ft^2 x 4 or 16 ft^2

The total area of the border would be 16 ft^2 + 12 ft^2 + 16 ft^2 or **44ft^2**

6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

6.G.2 Previously students calculated the volume of right rectangular prisms (boxes) using whole number edges. The use of models was emphasized as students worked to derive the formula V = Bh (5.MD.3, 5.MD.4, 5.MD.5) The unit cube was 1 x 1 x 1.
In 6th grade the unit cube will have fractional edge lengths. (ie. ½ • ½ • ½) Students find the volume of the right rectangular prism with these unit cubes.
Students need multiple opportunities to measure volume by filling rectangular prisms with blocks and looking at the relationship between the total volume and the area of the base. Through these experiences, students *derive* the volume formula (volume equals the area of the base times the height). Students can explore the connection between filling a box with unit cubes and the volume formula using interactive applets such as the Cubes Tool on NCTM’s Illuminations (http://illuminations.nctm.org/ActivityDetail.aspx?ID=6).

In addition to filling boxes, students can draw diagrams to represent fractional side lengths, connecting with multiplication of fractions. This process is similar to composing and decomposing two-dimensional shapes.

Example 1:

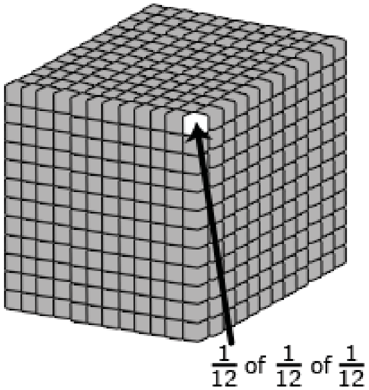
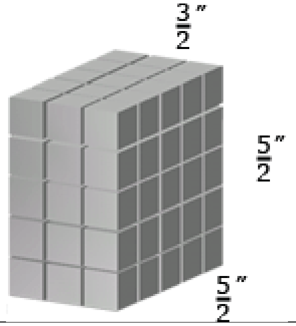
A right rectangular prism has edges of $1\frac{1}{4}$”, 1” and $1\frac{1}{2}$”. How many cubes with side lengths of $\frac{1}{4}$ would be needed to fill the prism? What is the volume of the prism?

Solution:

The number of $\frac{1}{4}$” cubes can be found by recognizing the smaller cubes would be $\frac{1}{4}$” on all edges, changing the dimensions to $\frac{5}{4}$”, $\frac{4}{4}$” and $\frac{6}{4}$”. The number of one-fourth inch unit cubes making up the prism is 120 (5 x 4 x 6).

Each smaller cube has a volume of $\frac{1}{64}$ ($\frac{1}{4}$” x $\frac{1}{4}$” x $\frac{1}{4}$”), meaning 64 small cubes would make up the unit cube.

Therefore, the volume is $\frac{5}{4}$ x $\frac{6}{4}$ x $\frac{4}{4}$ or $\frac{120}{64}$ (120 smaller cubes with volumes of $\frac{1}{64}$ or $1\frac{56}{64}$ →1 unit cube with 56 smaller cubes with a volume of $\frac{1}{64}$.

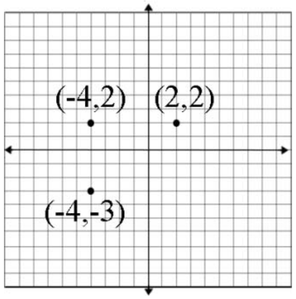
	Example 2: The model shows a cubic foot filled with cubic inches. The cubic inches can also be labeled as a fractional cubic unit with dimensions of $\frac{1}{12}$ft^{3}.  Example 3: The model shows a rectangular prism with dimensions $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{5}{2}$ inches. Each of the cubic units in the model is $\frac{1}{2}$in. on each side. Students work with the model to illustrate $\frac{3}{2}$x $\frac{5}{2}$x $\frac{5}{2}$ = (3 x 5 x 5) x $\frac{1}{8}$. Students reason that a small cube has volume of $\frac{1}{8}$ in^{3} because 8 of them fit in a unit cube. 
6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	**6.G.3** Students are given the coordinates of polygons to draw in the coordinate plane. If both *x*-coordinates are the same (2, -1) and (2, 4), then students recognize that a vertical line has been created and the distance between these coordinates is the distance between -1 and 4, or 5. If both the *y*-coordinates are the same (-5, 4) and (2, 4), then students recognize that a horizontal line has been created and the distance between these coordinates is the distance between -5 and 2, or 7. Using this understanding, student solve real-world and mathematical problems, including finding the area and perimeter of geometric figures drawn on a coordinate plane.

This standard can be taught in conjunction with **6.G.1** to help students develop the formula for the triangle by using the squares of the coordinate grid. Given a triangle, students can make the corresponding square or rectangle and realize the triangle is ½.

Students progress from counting the squares to making a rectangle and recognizing the triangle as ½ to the development of the formula for the area of a triangle.

Example 1:

If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle? Find the area and the perimeter of the rectangle.



Solution:

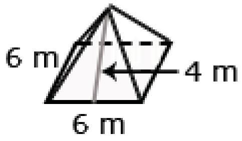
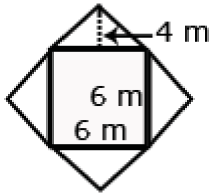
To determine the distance along the *x*-axis between the point (-4, 2) and (2, 2) a student must recognize that -4 is |-4| or 4 units to the left of 0 and 2 is |2| or 2 units to the right of zero, so the two points are total of 6 units apart along the x-axis. Students should represent this on the coordinate grid and numerically with an absolute value expression, |-4| + |2| . The length is 6 and the width is 5.

The fourth vertex would be (2, -3).
The area would be 5 x 6 or 30 units^2.
The perimeter would be 5 + 5 + 6 + 6 or 22 units.

Example 2:

On a map, the library is located at (-2, 2), the city hall building is located at (0,2), and the high school is located at (0,0). Represent the locations as points on a coordinate grid with a unit of 1 mile.

1. What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know?
2. What shape does connecting the three locations form? The city council is planning to place a city park in this area. How large is the area of the planned park?

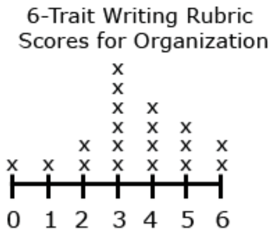
	Solution: 1. The distance from the library to city hall is 2 miles. The coordinates of these buildings have the same *y*-coordinate. The distance between the *x*-coordinates is 2 (from -2 to 0). 2. The three locations form a right triangle. The area is 2 mi^2.
6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	**6.G.4** A net is a two-dimensional representation of a three-dimensional figure. Students represent three-dimensional figures whose nets are composed of rectangles and triangles. Students recognize that parallel lines on a net are congruent. Using the dimensions of the individual faces, students calculate the area of each rectangle and/or triangle and add these sums together to find the surface area of the figure. Students construct models and nets of three-dimensional figures, describing them by the number of edges, vertices, and faces. Solids include rectangular and triangular prisms. Students are expected to use the net to calculate the surface area. Students can create nets of 3D figures with specified dimensions using the Dynamic Paper Tool on NCTM's Illuminations (http://illuminations.nctm.org/ActivityDetail.aspx?ID=205). Students also describe the types of faces needed to create a three-dimensional figure. Students make and test conjectures by determining what is needed to create a specific three-dimensional figure. Example 1: Describe the shapes of the faces needed to construct a rectangular pyramid. Cut out the shapes and create a model. Did your faces work? Why or why not? Example 2: Create the net for a given prism or pyramid, and then use the net to calculate the surface area.  

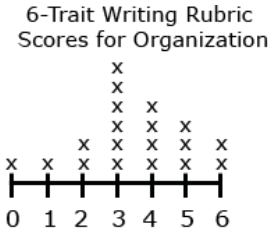
Statistics and Probability 6.SP

Common Core Cluster

Develop understanding of statistical variability.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **statistics, data, variability, distribution, dot plot, histograms, box plots, median, mean**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*	**6.SP.1** Statistics are numerical data relating to a group of individuals; statistics is also the name for the science of collecting, analyzing and interpreting such data. A statistical question anticipates an answer that varies from one individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources (i.e. documents). Students differentiate between statistical questions and those that are not. A statistical question is one that collects information that addresses differences in a population. The question is framed so that the responses will allow for the differences. For example, the question, "How tall am I?" is not a statistical question because there is only one response; however, the question, "How tall are the students in my class?" is a statistical question since the responses anticipates variability by providing a variety of possible anticipated responses that have numerical answers. Questions can result in a narrow or wide range of numerical values. Students might want to know about the fitness of the students at their school. Specifically, they want to know about the exercise habits of the students. So rather than asking "Do you exercise?" they should ask about the amount of exercise the students at their school get per week. A statistical question for this study could be: "How many hours per week on average do students at Jefferson Middle School exercise?"
6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center, spread, and overall shape.	**6.SP.2** The distribution is the arrangement of the values of a data set. Distribution can be described using center (median or mean), and spread. Data collected can be represented on graphs, which will show the shape of the distribution of the data. Students examine the distribution of a data set and discuss the center, spread and overall shape with dot plots, histograms and box plots. <u>Example 1:</u> The dot plot shows the writing scores for a group of students on organization. Describe the data. 

	Solution: The values range from 0 – 6. There is a peak at 3. The median is 3, which means 50% of the scores are greater than or equal to 3 and 50% are less than or equal to 3. The mean is 3.68. If all students scored the same, the score would be 3.68. NOTE: Mode as a measure of center and range as a measure of variability are not addressed in the CCSS and as such are not a focus of instruction. These concepts can be introduced during instruction as needed.
6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	**6.SP.3** Data sets contain many numerical values that can be summarized by one number such as a measure of center. The measure of center gives a numerical value to represent the center of the data (ie. midpoint of an ordered list or the balancing point). Another characteristic of a data set is the variability (or spread) of the values. Measures of variability are used to describe this characteristic. <u>Example 1</u>: Consider the data shown in the dot plot of the six trait scores for organization for a group of students. • How many students are represented in the data set? • What are the mean and median of the data set? What do these values mean? How do they compare? • What is the range of the data? What does this value mean?  *Solution:* • 19 students are represented in the data set. • The mean of the data set is 3.5. The median is 3. The mean indicates that is the values were equally distributed, all students would score a 3.5. The median indicates that 50% of the students scored a 3 or higher; 50% of the students scored a 3 or lower. • The range of the data is 6, indicating that the values vary 6 points between the lowest and highest scores.

Statistics and Probability 6.SP

Common Core Cluster

Summarize and describe distributions.

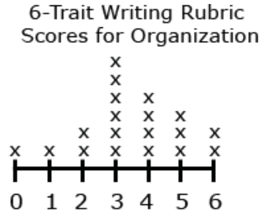
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **box plots, dot plots, histograms, frequency tables, cluster, peak, gap, mean, median, interquartile range, measures of center, measures of variability, data, Mean Absolute Deviation (M.A.D.), quartiles, lower quartile (1st quartile or Q_1), upper quartile (3rd quartile or Q_3), symmetrical, skewed, summary statistics, outlier**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
6.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	**6.SP.4** Students display data graphically using number lines. Dot plots, histograms and box plots are three graphs to be used. Students are expected to determine the appropriate graph as well as read data from graphs generated by others. Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers. A histogram shows the distribution of continuous data using intervals on the number line. The height of each bar represents the number of data values in that interval. In most real data sets, there is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful; however, a histogram can be used. Students group the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the bin changes the appearance of the graph and the conclusions may vary from it. A box plot shows the distribution of values in a data set by dividing the set into quartiles. It can be graphed either vertically or horizontally. The box plot is constructed from the five-number summary (minimum, lower quartile, median, upper quartile, and maximum). These values give a summary of the shape of a distribution. Students understand that the size of the box or whiskers represents the middle 50% of the data. Students can use applets to create data displays. Examples of applets include the Box Plot Tool and Histogram Tool on NCTM's Illuminations. Box Plot Tool - http://illuminations.nctm.org/ActivityDetail.aspx?ID=77 Histogram Tool -- http://illuminations.nctm.org/ActivityDetail.aspx?ID=78

Example 1:

Nineteen students completed a writing sample that was scored on organization. The scores for organization were 0, 1, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6. Create a data display. What are some observations that can be made from the data display?

Solution:



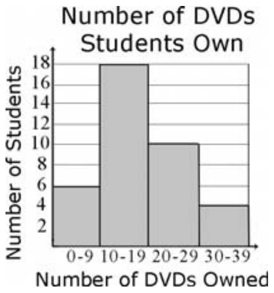
Example 2:

Grade 6 students were collecting data for a math class project. They decided they would survey the other two grade 6 classes to determine how many DVDs each student owns. A total of 48 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?

11	21	5	12	10	31	19	13	23	33
10	11	25	14	34	15	14	29	8	5
22	26	23	12	27	4	25	15	7	
2	19	12	39	17	16	15	28	16	

Solution:

A histogram using 5 intervals (bins) 0-9, 10-19, …30-39) to organize the data is displayed below.



Most of the students have between 10 and 19 DVDs as indicated by the peak on the graph. The data is pulled to the right since only a few students own more than 30 DVDs.

Example 3:

Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?

130	130	131	131	132	132	132	133	134	136
137	137	138	139	139	139	140	141	142	142
142	143	143	144	145	147	149	150		

Solution:

Five number summary

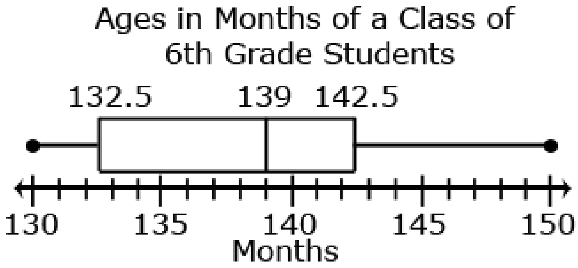
Minimum – 130 months

Quartile 1 (Q1) – (132 + 133) ÷ 2 = 132.5 months

Median (Q2) – 139 months

Quartile 3 (Q3) – (142 + 143) ÷ 2 = 142.5 months

Maximum – 150 months



This box plot shows that

- ¼ of the students in the class are from 130 to 132.5 months old
- ¼ of the students in the class are from 142.5 months to 150 months old
- ½ of the class are from 132.5 to 142.5 months old
- The median class age is 139 months.

6.SP.5 Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	**6.SP.5** Students summarize numerical data by providing background information about the attribute being measured, methods and unit of measurement, the context of data collection activities (addressing random sampling), the number of observations, and summary statistics. Summary statistics include quantitative measures of center (median and median) and variability (interquartile range and mean absolute deviation) including extreme values (minimum and maximum), mean, median, mode, range, and quartiles. Students record the number of observations. Using histograms, students determine the number of values between specified intervals. Given a box plot and the total number of data values, students identify the number of data points that are represented by the box. Reporting of the number of observations must consider the attribute of the data sets, including units (when applicable). Measures of Center Given a set of data values, students summarize the measure of center with the median or mean. The median is the value in the middle of a ordered list of data. This value means that 50% of the data is greater than or equal to it and that 50% of the data is less than or equal to it. The mean is the arithmetic average; the sum of the values in a data set divided by how many values there are in the data set. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students develop these understandings of what the mean represents by redistributing data sets to be level or fair (equal distribution) and by observing that the total distance of the data values above the mean is equal to the total distance of the data values below the mean (balancing point). Students use the concept of mean to solve problems. Given a data set represented in a frequency table, students calculate the mean. Students find a missing value in a data set to produce a specific average. Example 1: Susan has four 20-point projects for math class. Susan's scores on the first 3 projects are shown below: Project 1: 18 Project 2: 15 Project 3: 16 Project 4: ?? What does she need to make on Project 4 so that the average for the four projects is 17? Explain your reasoning. *Solution:* One possible solution to is calculate the total number of points needed (17 x 4 or 68) to have an average of 17. She has earned 49 points on the first 3 projects, which means she needs to earn 19 points on Project 4 (68 – 49 = 19).

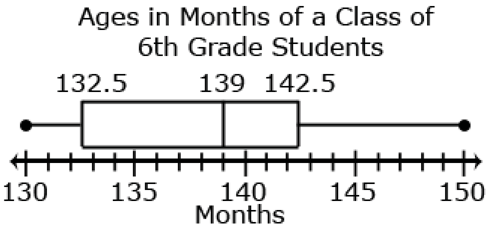
Measures of Variability

Measures of variability/variation can be described using the interquartile range or the Mean Absolute Deviation. The interquartile range (IQR) describes the variability between the middle 50% of a data set. It is found by subtracting the lower quartile from the upper quartile. It represents the length of the box in a box plot and is not affected by outliers.

Students find the IQR from a data set by finding the upper and lower quartiles and taking the difference or from reading a box plot.

Example 1:

What is the IQR of the data below:



Solution:

The first quartile is 132.5; the third quartile is 142.5. The IQR is 10 (142.5 – 132.5). This value indicates that the values of the middle 50% of the data vary by 10.

Mean Absolute Deviation (MAD) describes the variability of the data set by determining the absolute deviation (the distance) of each data piece from the mean and then finding the average of these deviations.

Both the interquartile range and the Mean Absolute Deviation are represented by a single numerical value. Higher values represent a greater variability in the data.

Example 2:

The following data set represents the size of 9 families:

3, 2, 4, 2, 9, 8, 2, 11, 4.

What is the MAD for this data set?

Solution:

The mean is 5. The MAD is the average variability of the data set. To find the MAD:

1. Find the deviation from the mean.
2. Find the absolute deviation for each of the values from step 1
3. Find the average of these absolute deviations.

The table below shows these calculations:

Data Value	**Deviation from Mean**	**Absolute Deviation**
3	-2	2
2	-3	3
4	-1	1
2	-3	3
9	4	4
8	3	3
2	-3	3
11	6	6
4	-1	1
MAD		26/9 = 2.89

This value indicates that on average family size varies 2.89 from the mean of 5.

Students understand how the measures of center and measures of variability are represented by graphical displays.

Students describe the context of the data, using the shape of the data and are able to use this information to determine an appropriate measure of center and measure of variability. The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median or middle value of the data set might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will be different, with the median frequently providing a better overall description of the data set.

We would like to acknowledge the Arizona Department of Education for allowing us to use some of their examples and graphics.

At A Glance

New to 7th Grade:

- Constant of proportionality (7.RP.2b)
- Percent of error (7.RP.3)
- Factoring to create equivalent expressions (7.EE.1)
- Triangle side lengths (7.G.2)
- Area and circumference of circles (7.G.4)
- Angles (supplementary, complementary, vertical) (7. G.5)
- Surface area and volume of pyramids (7.G.6)
- Probability (7.SP.5 – 7.SP.8)

Moved from 7th Grade:

- Similar and congruent polygons (moved to 8th grade)
- Surface area and volume of cylinders (moved to 8th grade – volume only)
- Creation of box plots and histograms (moved to 6th grade – 7th grade continues to compare)
- Linear relations and functions (y-intercept moved to 8th grade)
- Views from 3-Dimensional figures (removed from CCSS)
- Statistical measures (moved to 6th grade)

Notes:

- Topics may appear to be similar between the CCSS and the 2003 NCSCOS; however, the CCSS may be presented at a higher cognitive demand.
- Proportionality in 7th grade now includes identifying proportional relationships from tables and graphs; writing equations to represent proportional relationships.
- Using a number line for rational number operations is emphasized in CCSS.
- **For more detailed information, see the crosswalks (http://www.ncpublicschools.org/acre/standards/common-core-tools)**

Instructional considerations for CCSS implementation in 2012 – 2013:

- Work with ratio tables and relationships between tables, graphs and equations; focus on the multiplicative relationship between and within ratios (6.RP.3a, 6.RP.3b)
- Unit conversions within systems (6.RP.3d)
- Opposites and absolute value (6.NS.6a, 6.NS.7c)
- Distributive property with area models and factoring (6.EE.3) – prerequisite to 7.EE.1
- Volume of rectangular prisms (6.G.2) and surface area (6.G.4) – prerequisite to 7.G.6
- Mean Absolute Deviation (6.SP.5c) – prerequisite to 7.SP.3 and foundational to standard deviation in Math One

Standards for Mathematical Practice

The Common Core State Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that students complete.

Standards for Mathematical Practice	**Explanations and Examples**
1. Make sense of problems and persevere in solving them.	In grade 7, students solve problems involving ratios and rates and discuss how they solved the problems. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”.
2. Reason abstractly and quantitatively.	In grade 7, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others.	In grade 7, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). The students further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?”, “Does that always work?”. They explain their thinking to others and respond to others’ thinking.
4. Model with mathematics.	In grade 7, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students explore covariance and represent two quantities simultaneously. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences, make comparisons and formulate predictions. Students use experiments or simulations to generate data sets and create probability models. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to any problem’s context.
5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Students might use physical objects or applets to generate probability data and use graphing calculators or spreadsheets to manage and represent data in different forms.
6. Attend to precision.	In grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, specify units of measure, and label axes accurately. Students use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations or inequalities.

Standards for Mathematical Practice	**Explanations and Examples**
7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables making connections between the constant of proportionality in a table with the slope of a graph. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 3 (2 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality), $c = 6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving scale drawings, surface area, and volume. Students examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities.
8. Look for and express regularity in repeated reasoning.	In grade 7, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. They extend their thinking to include complex fractions and rational numbers. Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities. They create, explain, evaluate, and modify probability models to describe simple and compound events.

Grade 7 Critical Areas (from CCSS pg. 46)

The Critical Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction. The Critical Areas for seventh grade can be found on page 46 in the *Common Core State Standards for Mathematics.*

1. **Developing understanding of and applying proportional relationships**
   Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

2. **Developing understanding of operations with rational numbers and working with expressions and linear equations**
   Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

3. **Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume**
   Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

4. **Drawing inferences about populations based on samples**
   Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Ratios and Proportional Relationships 7.RP

Common Core Cluster

Analyze proportional relationships and use them to solve real-world and mathematical problems.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **unit rates, ratios, proportional relationships, proportions, constant of proportionality, complex fractions**

A detailed progression of the Ratios and Proportional Relationships domain with examples can be found at http://commoncoretools.wordpress.com/

<table>
<tr><th>Common Core Standard</th><th>Unpacking
What does this standard mean that a student will know and be able to do?</th></tr>
<tr><td>

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*

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7.RP.1 Students continue to work with unit rates from 6th grade; however, the comparison now includes fractions compared to fractions. The comparison can be with like or different units. Fractions may be proper or improper.

Example 1:

If $\frac{1}{2}$ gallon of paint covers $\frac{1}{6}$ of a wall, then how much paint is needed for the entire wall?

Solution:

$\frac{1}{2}$ gal / $\frac{1}{6}$ wall.

3 gallons per 1 wall

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<tr><td>

7.RP.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

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7.RP.2 Students' understanding of the multiplicative reasoning used with proportions continues from 6th grade. Students determine if two quantities are in a proportional relationship from a table. Fractions and decimals could be used with this standard.

Note: This standard focuses on the representations of proportions. Solving proportions is addressed in **7.SP.3**.

Example 1:

The table below gives the price for different numbers of books. Do the numbers in the table represent a proportional relationship?

<table>
<tr><th>Number of Books</th><th>Price</th></tr>
<tr><td>1</td><td>3</td></tr>
<tr><td>3</td><td>9</td></tr>
<tr><td>4</td><td>12</td></tr>
<tr><td>7</td><td>18</td></tr>
</table>

</td></tr>
</table>

Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*

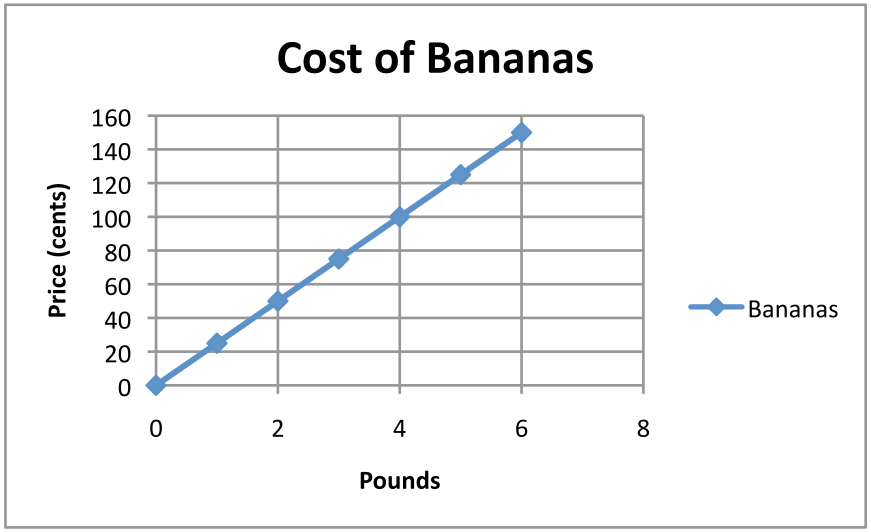
c. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

Solution:
Students can examine the numbers to determine that the price is the number of books multiplied by 3, except for 7 books. The row with seven books for $18 is not proportional to the other amounts in the table; therefore, the table does ***not*** represent a proportional relationship.

Students graph relationships to determine if two quantities are in a proportional relationship and to interpret the ordered pairs. If the amounts from the table above are graphed (number of books, price), the pairs (1, 3), (3, 9), and (4, 12) will form a straight line through the origin (0 books, 0 dollars), indicating that these pairs are in a proportional relationship. The ordered pair (4, 12) means that 4 books cost $12. However, the ordered pair (7, 18) would not be on the line, indicating that it is not proportional to the other pairs.

The ordered pair (1, 3) indicates that 1 book is $3, which is the unit rate. The y-coordinate when $x = 1$ will be the unit rate. The constant of proportionality is the unit rate. Students identify this amount from tables (see example above), graphs, equations and verbal descriptions of proportional relationships.

Example 2:
The graph below represents the price of the bananas at one store. What is the constant of proportionality?



Solution:
From the graph, it can be determined that 4 pounds of bananas is $1.00; therefore, 1 pound of bananas is $0.25, which is the constant of proportionality for the graph. Note: Any point on the line will yield this constant of proportionality.

Students write equations from context and identify the coefficient as the unit rate which is also the constant of proportionality.

Example 3:

The price of bananas at another store can be determined by the equation: $P = \$0.35n$, where P is the price and n is the number of pounds of bananas. What is the constant of proportionality (unit rate)?

Solution:

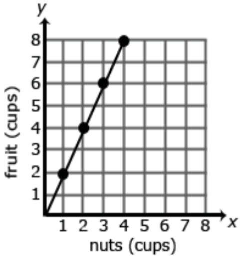
The constant of proportionality is the coefficient of x (or the independent variable). The constant of proportionality is 0.35.

Example 4:

A student is making trail mix. Create a graph to determine if the quantities of nuts and fruit are proportional for each serving size listed in the table. If the quantities are proportional, what is the constant of proportionality or unit rate that defines the relationship? Explain how the constant of proportionality was determined and how it relates to both the table and graph.

Serving Size	**1**	**2**	**3**	**4**
cups of nuts (x)	1	2	3	4
cups of fruit (y)	2	4	6	8

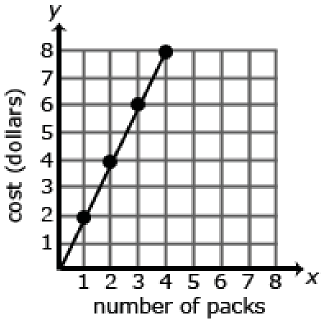
Solution:



The relationship is proportional. For each of the other serving sizes there are 2 cups of fruit for every 1 cup of nuts (2:1).

The constant of proportionality is shown in the first column of the table and by the steepness (rate of change) of the line on the graph.

Example 5:
The graph below represents the cost of gum packs as a unit rate of $2 dollars for every pack of gum. The unit rate is represented as $2/pack. Represent the relationship using a table and an equation.



Solution:
Table:

Number of Packs of Gum (g)	**Cost in Dollars (d)**
0	0
1	2
2	4
3	6
4	8

Equation: $d = 2g$, where d is the cost in dollars and g is the packs of gum

A common error is to reverse the position of the variables when writing equations. Students may find it useful to use variables specifically related to the quantities rather than using x and y. Constructing verbal models can also be helpful. A student might describe the situation as "the number of packs of gum times the cost for each pack is the total cost in dollars". They can use this verbal model to construct the equation. Students can check their equation by substituting values and comparing their results to the table. The checking process helps student revise and recheck their model as necessary. The number of packs of gum times the cost for each pack is the total cost.
($g \times 2 = d$)

Ratios and Proportional Relationships 7.RP

Common Core Cluster

Analyze proportional relationships and use them to solve real-world and mathematical problems.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **proportion, ratio, proportional relationships, percent, simple interest, rate, principal, tax, discount, markup, markdown, gratuity, commissions, fees, percent of error**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error*	**7.RP.3** In 6th grade, students used ratio tables and unit rates to solve problems. Students expand their understanding of proportional reasoning to solve problems that are easier to solve with cross-multiplication. Students understand the mathematical foundation for cross-multiplication. An explanation of this foundation can be found in Developing Effective Fractions Instruction for Kindergarten Through 8th Grade. Example 1: Sally has a recipe that needs $\frac{3}{4}$ teaspoon of butter for every 2 cups of milk. If Sally increases the amount of milk to 3 cups of milk, how many teaspoons of butter are needed? Using these numbers to find the unit rate may not be the most efficient method. Students can set up the following proportion to show the relationship between butter and milk. $\frac{\frac{3}{4}}{2} = \frac{x}{3}$ *Solution:* One possible solution is to recognize that $2 \bullet 1\frac{1}{2} = 3$ so $\frac{3}{4} \bullet 1\frac{1}{2} = x$. The amount of butter needed would be $1\frac{1}{8}$ teaspoons. A second way to solve this proportion is to use cross-multiplication $\frac{3}{4} \bullet 3 = 2x$. Solving for x would give $1\frac{1}{8}$ teaspoons of butter. Finding the percent error is the process of expressing the size of the error (or deviation) between two measurements. To calculate the percent error, students determine the absolute deviation (positive difference) between an actual measurement and the accepted value and then divide by the accepted value. Multiplying by 100 will give the percent error. (Note the similarity between percent error and percent of increase or decrease) $\% \text{ error} = \frac{\lvert \text{estimated value - actual value} \rvert}{\text{actual value}} \times 100\,\%$

Example 2:

Jamal needs to purchase a countertop for his kitchen. Jamal measured the countertop as 5 ft. The actual measurement is 4.5 ft. What is Jamal's percent error?

Solution:

$$\% \text{ error} = \frac{|5 \text{ ft} - 4.5 \text{ ft}|}{4.5} \text{ x } 100$$

$$\% \text{ error} = \frac{0.5 \text{ ft}}{4.5} \text{ x } 100$$

The use of proportional relationships is also extended to solve percent problems involving sales tax, markups and markdowns simple interest ($I = prt$, where I = interest, p = principal, r = rate, and t = time (in years)), gratuities and commissions, fees, percent increase and decrease, and percent error.

Students should be able to explain or show their work using a representation (numbers, words, pictures, physical objects, or equations) and verify that their answer is reasonable. Students use models to identify the parts of the problem and how the values are related. For percent increase and decrease, students identify the starting value, determine the difference, and compare the difference in the two values to the starting value.

For example, Games Unlimited buys video games for $10. The store increases their purchase price by 300%? What is the sales price of the video game?
Using proportional reasoning, if $10 is 100% then what amount would be 300%? Since 300% is 3 times 100%, $30 would be $10 times 3. Thirty dollars represents the amount of increase from $10 so the new price of the video game would be $40.

Example 3:

Gas prices are projected to increase by 124% by April 2015. A gallon of gas currently costs $3.80. What is the projected cost of a gallon of gas for April 2015?

Solution:

Possible response: "The original cost of a gallon of gas is $3.80. An increase of 100% means that the cost will double. Another 24% will need to be added to figure out the final projected cost of a gallon of gas. Since 25% of $3.80 is about $0.95, the projected cost of a gallon of gas should be around $8.15."

$$\$3.80 + 3.80 + (0.24 \bullet 3.80) = 2.24 \text{ x } 3.80 = \$8.15$$

100%	100%	24%
$3.80	$3.80	?

Example 4:
A sweater is marked down 33% off the original price. The original price was $37.50. What is the sale price of the sweater before sales tax?

Solution:
The discount is 33% times 37.50. The sale price of the sweater is the original price minus the discount or 67% of the original price of the sweater, or Sale Price = 0.67 x Original Price.

<table>
<tr><td colspan="2">37.50</td></tr>
<tr><td>33% of 37.50
Discount</td><td>67% of 37.50
Sale price of sweater</td></tr>
</table>

Example 5:
A shirt is on sale for 40% off. The sale price is $12. What was the original price? What was the amount of the discount?

Solution:

<table>
<tr><td>Discount
40% of original</td><td>Sale Price - $12
60% of original price</td></tr>
<tr><td colspan="2">Original Price (p)</td></tr>
</table>

The sale price is 60% of the original price. This reasoning can be expressed as $12 = 0.60p$. Dividing both sides by 0.60 gives an original price of $20.

Example 6:
At a certain store, 48 television sets were sold in April. The manager at the store wants to encourage the sales team to sell more TVs by giving all the sales team members a bonus if the number of TVs sold increases by 30% in May. How many TVs must the sales team sell in May to receive the bonus? Justify the solution.

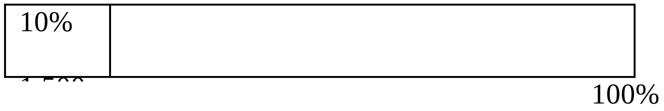
Solution:
The sales team members need to sell the 48 and an additional 30% of 48. 14.4 is exactly 30% so the team would need to sell 15 more TVs than in April or 63 total (48 + 15)

Example 7:

A salesperson set a goal to earn $2,000 in May. He receives a base salary of $500 per month as well as a 10% commission for all sales in that month. How much merchandise will he have to sell to meet his goal?

Solution:

$2,000 - $500 = $1,500 or the amount needed to be earned as commission. 10% of what amount will equal $1,500.



Because 100% is 10 times 10%, then the commission amount would be 10 time 1,500 or 15,000

Example 8:

After eating at a restaurant, Mr. Jackson’s bill before tax is $52.50 The sales tax rate is 8%. Mr. Jackson decides to leave a 20% tip for the waiter based on the pre-tax amount. How much is the tip Mr. Jackson leaves for the waiter? How much will the total bill be, including tax and tip? Express your solution as a multiple of the bill.

Solution:

The amount paid = $\underbrace{0.20 \text{ x } \$52.50}_{\text{tip}} + \underbrace{0.08 \text{ x } \$52.50}_{\text{tax}}$ = 0.28 x $52.50 or $14.70 for the tip and tax. The total bill would be $67.20,

Example 9:

Stephanie paid $9.18 for a pair of earrings. This amount includes a tax of 8%. What was the cost of the item before tax?

Solution:

One possible solution path follows:

$9.18 represents 100% of the cost of the earrings + 8% of the cost of the earrings. This representation can be expressed as 1.08c = 9.18, where c represents the cost of the earrings. Solving for c gives $8.50 for the cost of the earrings.

Several problem situations have been represented with this standard; however, every possible situation cannot be addressed here.

The Number System 7.NS

Common Core Cluster

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **rational numbers, integers, additive inverse**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.	**7.NS.1** Students add and subtract rational numbers. Visual representations may be helpful as students begin this work; they become less necessary as students become more fluent with these operations. The expectation of the CCSS is to build on student understanding of number lines developed in 6th grade. <u>Example 1:</u> Use a number line to add -5 + 7. *Solution:* Students find -5 on the number line and move 7 in a positive direction (to the right). The stopping point of 2 is the sum of this expression. Students also add negative fractions and decimals and interpret solutions in given contexts. In 6th grade, students found the distance of horizontal and vertical segments on the coordinate plane. In 7th grade, students build on this understanding to recognize subtraction is finding the distance between two numbers on a number line. In the example, 7 – 5, the difference is the distance between 7 and 5, or 2, in the direction of 5 to 7 (positive). Therefore the answer would be 2. <u>Example 2:</u> Use a number line to subtract: -6 – (-4) *Solution:* This problem is asking for the distance between -6 and -4. The distance between -6 and -4 is 2 and the direction from -4 to -6 is left or negative. The answer would be -2. Note that this answer is the same as adding the opposite of -4: -6 + 4 = -2

Example 3:

Use a number line to illustrate:

- $p – q$ ie. 7 – 4
- $p + (-q)$ ie. 7 + (– 4)
- Is this equation true $p – q = p + (-q)$?

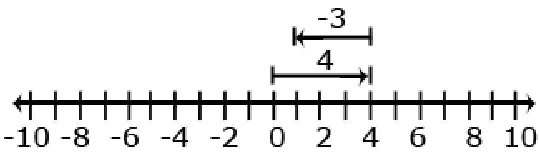
Students explore the above relationship when p is negative and q is positive and when both p and q are negative. Is this relationship always true?

Example 4:

Morgan has $4 and she needs to pay a friend $3. How much will Morgan have after paying her friend?

Solution:

4 + (-3) = 1 or (-3) + 4 = 1



7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q

7.NS.2 Students understand that multiplication and division of integers is an extension of multiplication and division of whole numbers. Students recognize that when division of rational numbers is represented with a fraction bar, each number can have a negative sign.

Example 1:

Which of the following fractions is equivalent to $\frac{-4}{5}$? Explain your reasoning.

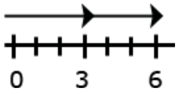
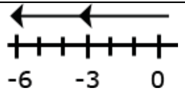
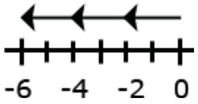
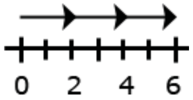
a. $\frac{4}{-5}$ b. $\frac{-16}{20}$ c. $\frac{-4}{-5}$

Example Set 2:

are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.
d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Examine the family of equations in the table below. What patterns are evident? Create a model and context for each of the products. Write and model the family of equations related to 3 x 4 = 12.

Equation	Number Line Model	Context
2 • 3 = 6		Selling two packages of apples at $3.00 per pack
2 • -3 = -6		Spending 3 dollars each on 2 packages of apples
-2 • 3 = -6		Owing 2 dollars to each of your three friends
-2 • -3 = 6		Forgiving 3 debts of $2.00 each

Using long division from elementary school, students understand the difference between terminating and repeating decimals. This understanding is foundational for the work with rational and irrational numbers in 8th grade.

Example 3:

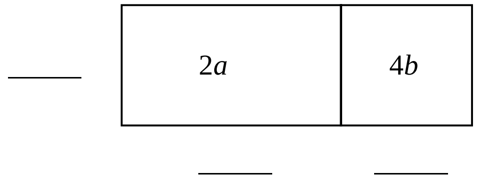
	Using long division, express the following fractions as decimals. Which of the following fractions will result in terminating decimals; which will result in repeating decimals? Identify which fractions will terminate (the denominator of the fraction in reduced form only has factors of 2 and/or 5)
7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.[1] [1] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.	**7.NS.3** Students use order of operations from 6th grade to write and solve problem with all rational numbers. <u>Example 1:</u> Calculate: [-10(-0.9)] – [(-10) • 0.11] *Solution:* 10.1 <u>Example 2:</u> Jim's cell phone bill is automatically deducting $32 from his bank account every month. How much will the deductions total for the year? *Solution:* -32 + (-32) + (-32) + (-32)+ (-32) + (-32) + (-32) + (-32) + (-32) + (-32) + (-32) + (-32) = 12 (-32) <u>Example 3:</u> It took a submarine 20 seconds to drop to 100 feet below sea level from the surface. What was the rate of the descent? *Solution:* $\frac{\text{-100 feet}}{\text{20 seconds}} = \frac{\text{- 5 feet}}{\text{1 second}} =$ -5 ft/sec <u>Example 4:</u> A newspaper reports these changes in the price of a stock over four days: $\frac{-1}{8}, \frac{-5}{8}, \frac{3}{8}, \frac{-9}{8}$. What is the average daily change? *Solution:* The sum is $\frac{-12}{8}$; dividing by 4 will give a daily average of $\frac{-3}{8}$

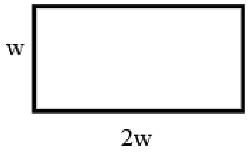
Expressions and Equations 7.EE

Common Core Cluster

Use properties of operations to generate equivalent expressions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **coefficients, like terms, distributive property, factor**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	**7.EE.1** This is a continuation of work from 6th grade using properties of operations (table 3, pg. 90) and combining like terms. Students apply properties of operations and work with rational numbers (integers and positive / negative fractions and decimals) to write equivalent expressions. Example 1: What is the length and width of the rectangle below?  *Solution:* The Greatest Common Factor (GCF) is 2, which will be the width because the width is in common to both rectangles. To get the area 2a multiply by a, which is the length of the first rectangles. To get the area of 4b, multiply by 2b, which will be the length of the second rectangle. The final answer will be 2(a + 2b) Example 2: Write an equivalent expression for 3(x + 5) – 2. *Solution:* $3x + 15 – 2$ Distribute the 3 $3x + 13$ Combine like terms

	Example 3: Suzanne says the two expressions $2(3a – 2) + 4a$ and $10a – 2$ are equivalent? Is she correct? Explain why or why not? *Solution:* The expressions are not equivalent. One way to prove this is to distribute and combine like terms in the first expression to get $10a – 4$, which is not equivalent to the second expression. A second explanation is to substitute a value for the variable and perform the calculations. For example, if 2 is substituted for a then the value of the first expression is 16 while the value of the second expression is 18. Example 4: Write equivalent expressions for: $3a + 12$. *Solution:* Possible solutions might include factoring as in $3(a + 4)$, or other expressions such as $a + 2a + 7 + 5$. Example 5: A rectangle is twice as long as its width. One way to write an expression to find the perimeter would be $w + w + 2w + 2w$. Write the expression in two other ways. *Solution:* $6w$ ***or*** $2(2w)$  Example 6: An equilateral triangle has a perimeter of $6x + 15$. What is the length of each side of the triangle? *Solution:* $3(2x + 5)$, therefore each side is $2x + 5$ units long.
7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*	**7.EE.2** Students understand the reason for rewriting an expression in terms of a contextual situation. For example, students understand that a 20% discount is the same as finding 80% of the cost, c $(0.80c)$. Example 1: All varieties of a certain brand of cookies are \$3.50. A person buys peanut butter cookies and chocolate chip cookies. Write an expression that represents the total cost, T, of the cookies if p represents the number of peanut butter cookies and c represents the number of chocolate chip cookies

Solution:

Students could find the cost of each variety of cookies and then add to find the total.

$T = 3.50p + 3.50c$

Or students could recognize that multiplying 3.50 by the total number of boxes (regardless of variety) will give the same total.

$T = 3.50(p + c)$

Example 2:

Jamie and Ted both get paid an equal hourly wage of $9 per hour. This week, Ted made an additional $27 dollars in overtime. Write an expression that represents the weekly wages of both if J = the number of hours that Jamie worked this week and T = the number of hours Ted worked this week? What is another way to write the expression?

Solution:

Students may create several different expressions depending upon how they group the quantities in the problem.

Possible student responses are:

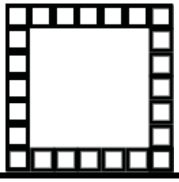
Response 1: To find the total wage, first multiply the number of hours Jamie worked by 9. Then, multiply the number of hours Ted worked by 9. Add these two values with the $27 overtime to find the total wages for the week. The student would write the expression $9J + 9T + 27$.

Response 2: To find the total wages, add the number of hours that Ted and Jamie worked. Then, multiply the total number of hours worked by 9. Add the overtime to that value to get the total wages for the week. The student would write the expression $9(J + T) + 27$.

Response 3: To find the total wages, find out how much Jamie made and add that to how much Ted made for the week. To figure out Jamie's wages, multiply the number of hours she worked by 9. To figure out Ted's wages, multiply the number of hours he worked by 9 and then add the $27 he earned in overtime. My final step would be to add Jamie and Ted wages for the week to find their combined total wages. The student would write the expression $(9J) + (9T + 27)$.

Example 3:

Given a square pool as shown in the picture, write four different expressions to find the total number of tiles in the border. Explain how each of the expressions relates to the diagram and demonstrate that the expressions are equivalent. Which expression is most useful? Explain.



Expressions and Equations 7.EE

Common Core Cluster

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **numeric expressions, algebraic expressions, maximum, minimum**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*	**7.EE.3** Students solve contextual problems and mathematical problems using rational numbers. Students convert between fractions, decimals, and percents as needed to solve the problem. Students use estimation to justify the reasonableness of answers. <u>Example 1:</u> Three students conduct the same survey about the number of hours people sleep at night. The results of the number of people who sleep 8 hours a nights are shown below. In which person's survey did the most people sleep 8 hours? • Susan reported that 18 of the 48 people she surveyed get 8 hours sleep a night • Kenneth reported that 36% of the people he surveyed get 8 hours sleep a night • Jamal reported that 0.365 of the people he surveyed get 8 hours sleep a night *Solution:* In Susan's survey, the number is 37.5%, which is the greatest percentage. Estimation strategies for calculations with fractions and decimals extend from students' work with whole number operations. Estimation strategies include, but are not limited to: • front-end estimation with adjusting (using the highest place value and estimating from the front end making adjustments to the estimate by taking into account the remaining amounts), • clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate), • rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values), • using friendly or compatible numbers such as factors (students seek to fit numbers together - i.e., rounding to factors and grouping numbers together that have round sums like 100 or 1000), and • using benchmark numbers that are easy to compute (students select close whole numbers for fractions or decimals to determine an estimate).

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

7.EE.4a and b Students write an equation or inequality to model the situation. Students explain how they determined whether to write an equation or inequality and the properties of the real number system that you used to find a solution. In contextual problems, students define the variable and use appropriate units.

7.EE.4a

Students solve multi-step equations derived from word problems. Students use the arithmetic from the problem to generalize an algebraic solution

Example 1:

The youth group is going on a trip to the state fair. The trip costs $52. Included in that price is $11 for a concert ticket and the cost of 2 passes, one for the rides and one for the game booths. Each of the passes cost the same price. Write an equation representing the cost of the trip and determine the price of one pass.

Solution:

x =cost of one pass

x	x	11
52		

$2x + 11 = 52$

$2x = 41$

$x = \$20.50$

Example 2:

Solve: $\frac{2}{3}x – 4 = -16$

Solution:

$\frac{2}{3}x – 4 = -16$

$\frac{2}{3}x = -12$ Added 4 to both sides

$\frac{3}{2} \bullet \frac{2}{3}x = -12 \bullet \frac{3}{2}$ Multiply both sides by $\frac{3}{2}$

$x = -18$

Students could also reason that if $\frac{2}{3}$ of some amount is -12 then $\frac{1}{3}$ is -6. Therefore, the whole amount must be 3 times -6 or -18.

Example 3:
Amy had $26 dollars to spend on school supplies. After buying 10 pens, she had $14.30 left. How much did each pen cost including tax?

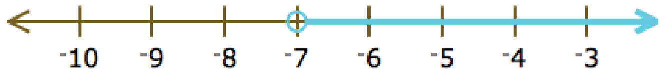
Solution:
x = number of pens
$26 = 14.30 + 10x$
Solving for x gives $1.17 for each pen.

Example 4:
The sum of three consecutive even numbers is 48. What is the smallest of these numbers?
Solution:
x = the smallest even number
$x + 2$ = the second even number
$x + 4$ = the third even number

$x + x + 2 + x + 4 = 48$
$3x + 6 = 48$
$3x = 42$
$x = 14$

Example 5:
Solve: $\frac{x + 3}{-2} = -5$

Solution:
$x = 7$

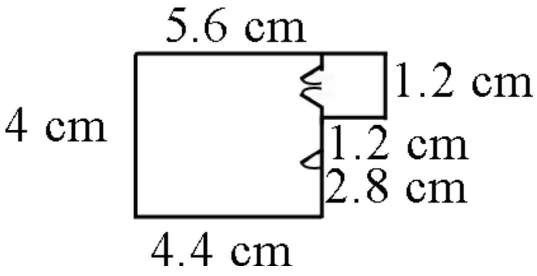
b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*	Students solve and graph inequalities and make sense of the inequality in context. Inequalities may have negative coefficients. Problems can be used to find a maximum or minimum value when in context. Example 1: Florencia has at most $60 to spend on clothes. She wants to buy a pair of jeans for $22 dollars and spend the rest on t-shirts. Each t-shirt costs $8. Write an inequality for the number of t-shirts she can purchase. *Solution:* x = cost of one t-shirt $8x + 22 \leq 60$ $x = 4.75$ → 4 is the most t-shirts she can purchase Example 2: Steven has $25 dollars to spend. He spent $10.81, including tax, to buy a new DVD. He needs to save $10.00 but he wants to buy a snack. If peanuts cost $0.38 per package including tax, what is the maximum number of packages that Steven can buy? *Solution:* x = number of packages of peanuts $25 \geq 10.81 + 10.00 + 0.38x$ $x = 11.03$ → Steven can buy 11 packages of peanuts Example 3: $7 – x > 5.4$ *Solution:* $x < 1.6$ Example 4: Solve -0.5x – 5 < -1.5 and graph the solution on a number line. *Solution:* $x > -7$ 

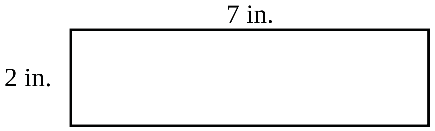
Geometry 7.G

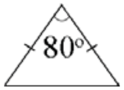
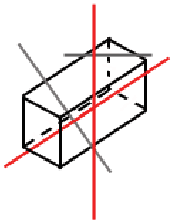
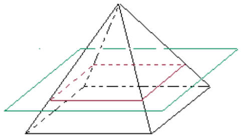
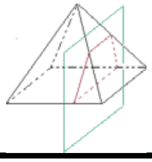
Common Core Cluster

Draw, construct, and describe geometrical figures and describe the relationships between them.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **scale drawing, dimensions, scale factor, plane sections, right rectangular prism, right rectangular pyramids, parallel, perpendicular**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	**7.G.1** Students determine the dimensions of figures when given a scale and identify the impact of a scale on actual length (one-dimension) and area (two-dimensions). Students identify the scale factor given two figures. Using a given scale drawing, students reproduce the drawing at a different scale. Students understand that the lengths will change by a factor equal to the product of the magnitude of the two size transformations. Example 1: Julie shows the scale drawing of her room below. If each 2 cm on the scale drawing equals 5 ft, what are the actual dimensions of Julie's room? Reproduce the drawing at 3 times its current size.  *Solution:* 5.6 cm → 14 ft 1.2 cm → 3 ft 2.8 cm → 7 ft 4.4 cm → 11 ft 4 cm → 10 ft

	Example 2: If the rectangle below is enlarged using a scale factor of 1.5, what will be the perimeter and area of the new rectangle?  *Solution*: The perimeter is linear or one-dimensional. Multiply the perimeter of the given rectangle (18 in.) by the scale factor (1.5) to give an answer of 27 in. Students could also increase the length and width by the scale factor of 1.5 to get 10.5 in. for the length and 3 in. for the width. The perimeter could be found by adding 10.5 + 10.5 + 3 + 3 to get 27 in. The area is two-dimensional so the scale factor must be squared. The area of the new rectangle would be 14 x 1.5^2 or 31.5 in^2.
7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	**7.G.2** Students draw geometric shapes with given parameters. Parameters could include parallel lines, angles, perpendicular lines, line segments, etc. Example 1: Draw a quadrilateral with one set of parallel sides and no right angles. Students understand the characteristics of angles and side lengths that create a unique triangle, more than one triangle or no triangle. Example 2: Can a triangle have more than one obtuse angle? Explain your reasoning. Example 3: Will three sides of any length create a triangle? Explain how you know which will work. Possibilities to examine are: a. 13 cm, 5 cm, and 6 cm b. 3 cm, 3cm, and 3 cm c. 2 cm, 7 cm, 6 cm *Solution:* “A” above will not work; “B” and “C” will work. Students recognize that the sum of the two smaller sides must be larger than the third side. Example 4: Is it possible to draw a triangle with a 90° angle and one leg that is 4 inches long and one leg that is 3 inches long? If so, draw one. Is there more than one such triangle? (NOTE: Pythagorean Theorem is NOT expected – this is an exploration activity only)

	<u>Example 5:</u> Draw a triangle with angles that are 60 degrees. Is this a unique triangle? Why or why not? <u>Example 6:</u> Draw an isosceles triangle with only one 80° angle. Is this the only possibility or can another triangle be drawn that will meet these conditions?  Through exploration, students recognize that the sum of the angles of any triangle will be 180°.
7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	**7.G.3** Students need to describe the resulting face shape from cuts made parallel and perpendicular to the bases of right rectangular prisms and pyramids. Cuts made parallel will take the shape of the base; cuts made perpendicular will take the shape of the lateral (side) face. Cuts made at an angle through the right rectangular prism will produce a parallelogram;   If the pyramid is cut with a plane (green) parallel to the base, the intersection of the pyramid and the plane is a square cross section (red).  If the pyramid is cut with a plane (green) passing through the top vertex and perpendicular to the base, the intersection of the pyramid and the plane is a triangular cross section (red).  If the pyramid is cut with a plane (green) perpendicular to the base, but not through the top vertex, the intersection of the pyramid and the plane is a trapezoidal cross section (red). http://intermath.coe.uga.edu/dictnary/descript.asp?termID=95

Geometry 7.G

Common Core Cluster

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **inscribed, circumference, radius, diameter, pi, π, supplementary, vertical, adjacent, complementary, pyramids, face, base**

Common Core Standard

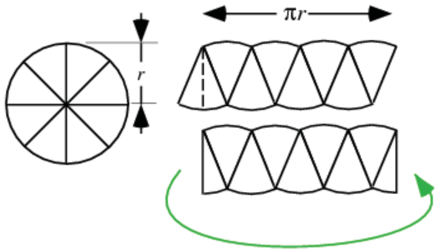
7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Unpacking

What does this standard mean that a student will know and be able to do?

7.G.4 Students understand the relationship between radius and diameter. Students also understand the ratio of circumference to diameter can be expressed as pi. Building on these understandings, students generate the formulas for circumference and area.

The illustration shows the relationship between the circumference and area. If a circle is cut into wedges and laid out as shown, a parallelogram results. Half of an end wedge can be moved to the other end a rectangle results. The height of the rectangle is the same as the radius of the circle. The base length is $\frac{1}{2}$ the circumference ($2\pi r$). The area of the rectangle (and therefore the circle) is found by the following calculations:



http://mathworld.wolfram.com/Circle.html

A_{rect} = Base x Height
Area = ½ ($2\pi r$) x r
Area = πr x r
Area = πr^2

Students solve problems (mathematical and real-world) involving circles or semi-circles.
Note: Because pi is an irrational number that neither repeats nor terminates, the measurements are approximate when 3.14 is used in place of π.

Example 1:

The seventh grade class is building a mini-golf game for the school carnival. The end of the putting green will be a circle. If the circle is 10 feet in diameter, how many square feet of grass carpet will they need to buy to cover the circle? How might someone communicate this information to the salesperson to make sure he receives a piece of carpet that is the correct size? Use 3.14 for pi.

Solution:

Area = πr^2
Area = 3.14 $(5)^2$
Area = 78.5 ft^2
To communicate this information, ask for a 9 ft by 9 ft square of carpet.

Example 2:

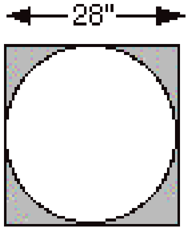
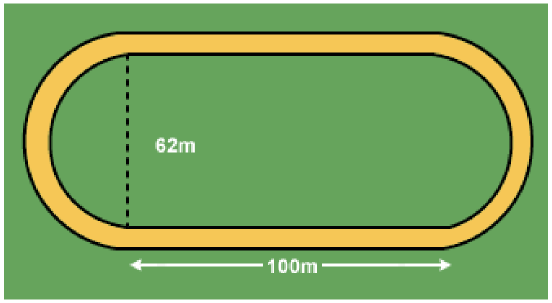
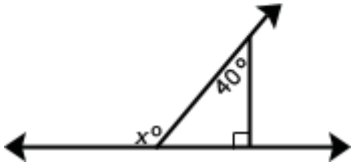
The center of the circle is at (5, -5). What is the area of the circle?

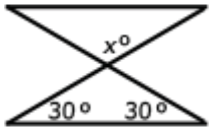
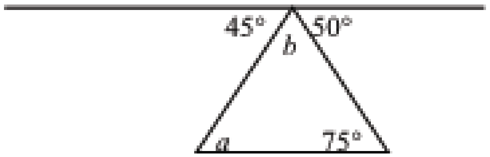
$$\frac{1}{2}$$

Solution:

The radius of the circle is 4. Using the formula, Area = πr^2, the area of the circle is approximately 50.24 units^2.

Students build on their understanding of area from 6th grade to find the area of left-over materials when circles are cut from squares and triangles or when squares and triangles are cut from circles.

	<u>Example 3:</u> If a circle is cut from a square piece of plywood, how much plywood would be left over?  *Solution:* The area of the square is 28 x 28 or 784 in^2. The diameter of the circle is equal to the length of the side of the square, or 28", so the radius would be 14". The area of the circle would be approximately 615.44 in2. The difference in the amounts (plywood left over) would be 168.56 in^2 (784 – 615.44). <u>Example 4:</u> What is the perimeter of the inside of the track.  *Solution:* The ends of the track are two semicircles, which would form one circle with a diameter of 62m. The circumference of this part would be 194.68 m. Add this to the two lengths of the rectangle and the perimeter is 2194.68 m "Know the formula" does not mean memorization of the formula. To "know" means to have an understanding of ***why*** the formula works and how the formula relates to the measure (area and circumference) and the figure. This understanding should be for *all* students.
7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	**7.G.5** Students use understandings of angles and deductive reasoning to write and solve equations <u>Example1:</u> Write and solve an equation to find the measure of angle *x*.  *Solution:* Find the measure of the missing angle inside the triangle (180 – 90 – 40), or 50°. The measure of angle *x* is supplementary to 50°, so subtract 50 from 180 to get a measure of 130° for *x*.

	Example 2: Find the measure of angle *x*.  *Solution:* First, find the missing angle measure of the bottom triangle (180 – 30 – 30 = 120). Since the 120 is a vertical angle to *x*, the measure of *x* is also 120°. Example 3: Find the measure of angle *b*.  Note: Not drawn to scale. *Solution:* Because, the 45°, 50° angles and *b* form are supplementary angles, the measure of angle *b* would be 85°. The measures of the angles of a triangle equal 180° so 75° + 85°+ *a* = 180°. The measure of angle *a* would be 20°.
7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	**7.G.6** Students continue work from 5th and 6th grade to work with area, volume and surface area of two-dimensional and three-dimensional objects. (composite shapes) Students will not work with cylinders, as circles are not polygons. At this level, students determine the dimensions of the figures given the area or volume. “Know the formula” does not mean memorization of the formula. To “know” means to have an understanding of ***why*** the formula works and how the formula relates to the measure (area and volume) and the figure. This understanding should be for *all* students. Surface area formulas are not the expectation with this standard. Building on work with nets in the 6th grade, students should recognize that finding the area of each face of a three-dimensional figure and adding the areas will give the surface area. No nets will be given at this level; however, students could create nets to aid in surface area calculations.

Students understanding of volume can be supported by focusing on the area of base times the height to calculate volume.
Students solve for missing dimensions, given the area or volume.

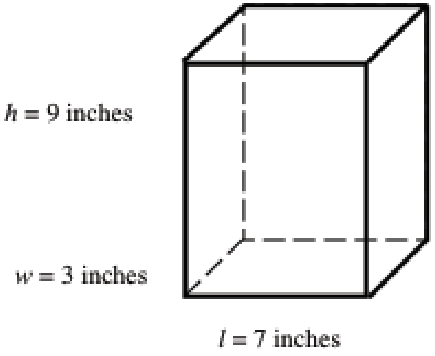
Example 2:
A triangle has an area of 6 square feet. The height is four feet. What is the length of the base?

Solution:
One possible solution is to use the formula for the area of a triangle and substitute in the known values, then solve for the missing dimension. The length of the base would be 3 feet.

Example 3:
The surface area of a cube is 96 in^2. What is the volume of the cube?

Solution:
The area of each face of the cube is equal. Dividing 96 by 6 gives an area of 16 in^2 for each face. Because each face is a square, the length of the edge would be 4 in. The volume could then be found by multiplying 4 x 4 x 4 or 64 in^3.

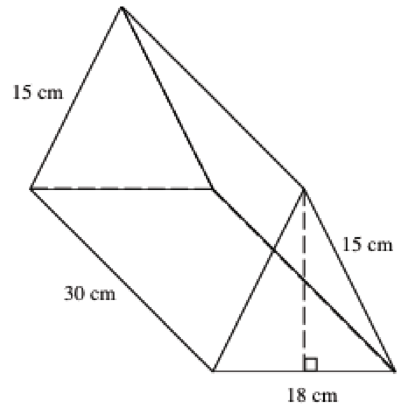
Example 4:
Huong covered the box to the right with sticky-backed decorating paper. The paper costs 3¢ per square inch. How much money will Huong need to spend on paper?



Solution:
The surface area can be found by using the dimensions of each face to find the area and multiplying by 2:
Front: 7 in. x 9 in. = 63 in^2 x 2 = 126 in^2
Top: 3 in. x 7 in. = 21 in^2 x 2 = 42 in^2
Side: 3 in. x 9 in. = 27 in^2 x 2 = 54 in^2

The surface area is the sum of these areas, or 222 in^2. If each square inch of paper cost \$0.03, the cost would be \$6.66.

Example 5:
Jennie purchased a box of crackers from the deli. The box is in the shape of a triangular prism (see diagram below). If the volume of the box is 3,240 cubic centimeters, what is the height of the triangular face of the box? How much packaging material was used to construct the cracker box? Explain how you got your answer.



Solution:
Volume can be calculated by multiplying the area of the base (triangle) by the height of the prism. Substitute given values and solve for the area of the triangle
$V = Bh$
$3{,}240 \text{ cm}^3 = B\,(30\text{cm})$

$$\frac{3{,}240 \text{ cm}^3}{30 \text{ cm}} = \frac{B(30\text{cm})}{30 \text{ cm}}$$

$108 \text{ cm}^2 = B$ (area of the triangle)

To find the height of the triangle, use the area formula for the triangle, substituting the known values in the formula and solving for height. The height of the triangle is 12 cm.

The problem also asks for the surface area of the package. Find the area of each face and add:
2 triangular bases: ½ (18 cm)(12 cm) = 108 cm^2 x 2 = 216 cm^2
2 rectangular faces: 15 cm x 30 cm = 450 cm^2 x 2 = 900 cm^2
1 rectangular face: 18 cm x 30 cm = 540 cm^2

Adding 216 cm^2 + 900 cm^2 + 540 cm^2 gives a total surface area of 1656 cm^2.

Statistics and Probability 7.SP

Common Core Cluster

Use random sampling to draw inferences about a population.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **random sampling, population, representative sample, inferences**

<table>
<tr>
<th>Common Core Standard</th>
<th>Unpacking
What does this standard mean that a student will know and be able to do?</th>
</tr>
<tr>
<td>7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.</td>
<td>7.SP.1 Students recognize that it is difficult to gather statistics on an entire population. Instead a random sample can be representative of the total population and will generate valid predictions. Students use this information to draw inferences from data. A random sample must be used in conjunction with the population to get accuracy. For example, a random sample of elementary students cannot be used to give a survey about the prom.

<u>Example 1:</u>
The school food service wants to increase the number of students who eat hot lunch in the cafeteria. The student council has been asked to conduct a survey of the student body to determine the students' preferences for hot lunch. They have determined two ways to do the survey. The two methods are listed below. Determine if each survey option would produce a random sample. Which survey option should the student council use and why?

1. Write all of the students' names on cards and pull them out in a draw to determine who will complete the survey.
2. Survey the first 20 students that enter the lunchroom.
3. Survey every 3rd student who gets off a bus.</td>
</tr>
<tr>
<td>7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. <i>For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.</i></td>
<td>7.SP.2 Students collect and use multiple samples of data to make generalizations about a population. Issues of variation in the samples should be addressed.

<u>Example 1:</u>
Below is the data collected from two random samples of 100 students regarding student's school lunch preference. Make at least two inferences based on the results.

<table>
<tr><th>Student Sample</th><th>Hamburgers</th><th>Tacos</th><th>Pizza</th><th>Total</th></tr>
<tr><td>#1</td><td>12</td><td>14</td><td>74</td><td>100</td></tr>
<tr><td>#2</td><td>12</td><td>11</td><td>77</td><td>100</td></tr>
</table>
<i>Solution:</i>
• Most students prefer pizza.
• More people prefer pizza and hamburgers and tacos combined.</td>
</tr>
</table>

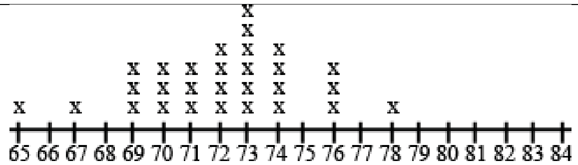
Statistics and Probability 7.SP

Common Core Cluster

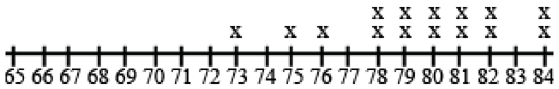
Draw informal comparative inferences about two populations.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **variation/variability, distribution, measures of center, measures of variability**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*	**7.SP.3** This is the students' first experience with comparing two data sets. Students build on their understanding of graphs, mean, median, Mean Absolute Deviation (MAD) and interquartile range from 6th grade. Students understand that 1. a full understanding of the data requires consideration of the measures of variability as well as mean or median, 2. variability is responsible for the overlap of two data sets and that an increase in variability can increase the overlap, and 3. median is paired with the interquartile range and mean is paired with the mean absolute deviation . Example: Jason wanted to compare the mean height of the players on his favorite basketball and soccer teams. He thinks the mean height of the players on the basketball team will be greater but doesn't know how much greater. He also wonders if the variability of heights of the athletes is related to the sport they play. He thinks that there will be a greater variability in the heights of soccer players as compared to basketball players. He used the rosters and player statistics from the team websites to generate the following lists. Basketball Team – Height of Players in inches for 2010 Season 75, 73, 76, 78, 79, 78, 79, 81, 80, 82, 81, 84, 82, 84, 80, 84 Soccer Team – Height of Players in inches for 2010 73, 73, 73, 72, 69, 76, 72, 73, 74, 70, 65, 71, 74, 76, 70, 72, 71, 74, 71, 74, 73, 67, 70, 72, 69, 78, 73, 76, 69 To compare the data sets, Jason creates a two dot plots on the same scale. The shortest player is 65 inches and the tallest players are 84 inches.



Height of Soccer Players (in)



Height of Basketball Players (in)

In looking at the distribution of the data, Jason observes that there is some overlap between the two data sets. Some players on both teams have players between 73 and 78 inches tall. Jason decides to use the mean and mean absolute deviation to compare the data sets.

The mean height of the basketball players is 79.75 inches as compared to the mean height of the soccer players at 72.07 inches, a difference of 7.68 inches.

The mean absolute deviation (MAD) is calculated by taking the mean of the absolute deviations for each data point. The difference between each data point and the mean is recorded in the second column of the table The difference between each data point and the mean is recorded in the second column of the table. Jason used rounded values (80 inches for the mean height of basketball players and 72 inches for the mean height of soccer players) to find the differences. The absolute deviation, absolute value of the deviation, is recorded in the third column. The absolute deviations are summed and divided by the number of data points in the set.

The mean absolute deviation is 2.14 inches for the basketball players and 2.53 for the soccer players. These values indicate moderate variation in both data sets.

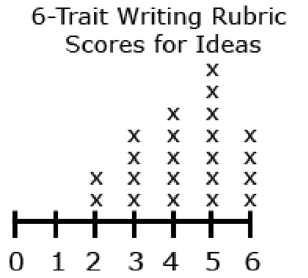
Solution:
There is slightly more variability in the height of the soccer players. The difference between the heights of the teams (7.68) is approximately 3 times the variability of the data sets (7.68 ÷ 2.53 = 3.04; 7.68 ÷ 2.14 = 3.59).

Soccer Players (*n* = 29)		
Height (in)	Deviation from Mean (in)	Absolute Deviation (in)
65	-7	7
67	-5	5
69	-3	3
69	-3	3
69	-3	3
70	-2	2
70	-2	2
71	-1	1
71	-1	1
71	-1	1
72	0	0
72	0	0
72	0	0
72	0	0
73	+1	1
73	+1	1
73	+1	1
73	+1	1
73	+1	1
73	+1	1
74	+2	2
74	+2	2
74	+2	2
74	+2	2
76	+4	4
76	+4	4
76	+4	4
78	+6	6
∑ = 2090		∑ = 62

Basketball Players (n = 16)		
Height (in)	Deviation from Mean (in)	Absolute Deviation (in)
73	-7	7
75	-5	5
76	-4	4
78	-2	2
78	-2	2
79	-1	1
79	-1	1
80	0	0
80	0	0
81	+1	1
81	+1	1
82	+2	2
82	+2	2
84	+4	4
84	+4	4
84	+4	4
∑ = 1276		∑ = 40

Mean = 2090 ÷ 29 =72 inches
MAD = 62 ÷ 29 = 2.14 inches

Mean = 1276 ÷ 16 =80 inches
MAD = 40 ÷ 16 = 2.53 inches

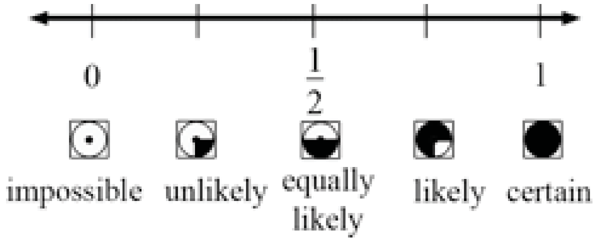
7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*	**7.SP.4** Students compare two sets of data using measures of center (mean and median) and variability MAD and IQR). Showing the two graphs vertically rather than side by side helps students make comparisons. For example, students would be able to see from the display of the two graphs that the ideas scores are generally higher than the organization scores. One observation students might make is that the scores for organization are clustered around a score of 3 whereas the scores for ideas are clustered around a score of 5.   <u>Example 1:</u> The two data sets below depict random samples of the management salaries in two companies. Based on the salaries below which measure of center will provide the most accurate estimation of the salaries for each company? • Company A: 1.2 million, 242,000, 265,500, 140,000, 281,000, 265,000, 211,000 • Company B: 5 million, 154,000, 250,000, 250,000, 200,000, 160,000, 190,000 *Solution:* The median would be the most accurate measure since both companies have one value in the million that is far from the other values and would affect the mean.

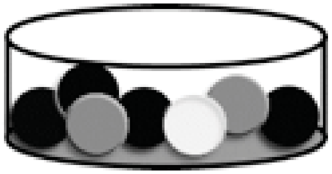
Statistics and Probability 7.SP

Common Core Cluster

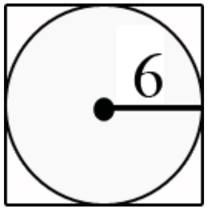
Investigate chance processes and develop, use, and evaluate probability models.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **sample spaces**
See list from essential standards work.

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	**7.SP.5** This is the students' first formal introduction to probability. Students recognize that the probability of any single event can be can be expressed in terms such as impossible, unlikely, likely, or certain or as a number between 0 and 1, inclusive, as illustrated on the number line below.  The closer the fraction is to 1, the greater the probability the event will occur. Larger numbers indicate greater likelihood. For example, if someone has 10 oranges and 3 apples, you have a greater likelihood of selecting an orange at random. Students also recognize that the sum of all possible outcomes is 1. <u>Example 1:</u> There are three choices of jellybeans – grape, cherry and orange. If the probability of getting a grape is $\frac{3}{10}$ and the probability of getting cherry is $\frac{1}{5}$, what is the probability of getting orange?

	Solution: The combined probabilities must equal 1. The combined probability of grape and cherry is $\frac{5}{10}$. The probability of orange must equal $\frac{5}{10}$ to get a total of 1. <u>Example 2:</u> The container below contains 2 gray, 1 white, and 4 black marbles. Without looking, if Eric chooses a marble from the container, will the probability be closer to 0 or to 1 that Eric will select a white marble? A gray marble? A black marble? Justify each of your predictions.  *Solution:* White marble: Closer to 0 Gray marble: Closer to 0 Black marble: Closer to 1 Students can use simulations such as Marble Mania on AAAS or the Random Drawing Tool on NCTM's Illuminations to generate data and examine patterns. Marble Mania http://www.sciencenetlinks.com/interactives/marble/marblemania.html Random Drawing Tool - http://illuminations.nctm.org/activitydetail.aspx?id=67
7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*	**7.SP.6** Students collect data from a probability experiment, recognizing that as the number of trials increase, the experimental probability approaches the theoretical probability. The focus of this standard is relative frequency -- The relative frequency is the observed number of successful events for a finite sample of trials. Relative frequency is the observed proportion of successful event, expressed as the value calculated by dividing the number of times an event occurs by the total number of times an experiment is carried out. <u>Example 1</u>: Suppose we toss a coin 50 times and have 27 heads and 23 tails. We define a head as a success. The relative frequency of heads is: $\frac{27}{50} = 54\%$ The probability of a head is 50%. The difference between the relative frequency of 54% and the probability of 50% is due to small sample size. The probability of an event can be thought of as its long-run relative frequency when the experiment is carried out many times.

	Students can collect data using physical objects or graphing calculator or web-based simulations. Students can perform experiments multiple times, pool data with other groups, or increase the number of trials in a simulation to look at the long-run relative frequencies. <u>Example 2:</u> Each group receives a bag that contains 4 green marbles, 6 red marbles, and 10 blue marbles. Each group performs 50 pulls, recording the color of marble drawn and replacing the marble into the bag before the next draw. Students compile their data as a group and then as a class. They summarize their data as experimental probabilities and make conjectures about theoretical probabilities (How many green draws would are expected if 1000 pulls are conducted? 10,000 pulls?). Students create another scenario with a different ratio of marbles in the bag and make a conjecture about the outcome of 50 marble pulls with replacement. (An example would be 3 green marbles, 6 blue marbles, 3 blue marbles.) Students try the experiment and compare their predictions to the experimental outcomes to continue to explore and refine conjectures about theoretical probability. <u>Example 3:</u> A bag contains 100 marbles, some red and some purple. Suppose a student, without looking, chooses a marble out of the bag, records the color, and then places that marble back in the bag. The student has recorded 9 red marbles and 11 purple marbles. Using these results, predict the number of red marbles in the bag. (Adapted from SREB publication *Getting Students Ready for Algebra I: What Middle Grades Students Need to Know and Be Able to Do)*
7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a*	**7.SP.7** Probabilities are useful for predicting what will happen over the long run. Using theoretical probability, students predict frequencies of outcomes. Students recognize an appropriate design to conduct an experiment with simple probability events, understanding that the experimental data give realistic estimates of the probability of an event but are affected by sample size. Students need multiple opportunities to perform probability experiments and compare these results to theoretical probabilities. Critical components of the experiment process are making predictions about the outcomes by applying the principles of theoretical probability, comparing the predictions to the outcomes of the experiments, and replicating the experiment to compare results. Experiments can be replicated by the same group or by compiling class data. Experiments can be conducted using various random generation devices including, but not limited to, bag pulls, spinners, number cubes, coin toss, and colored chips. Students can collect data using physical objects or graphing calculator or web-based simulations. Students can also develop models for geometric probability (i.e. a target).

girl will be selected. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*	<ins>Example 1:</ins> If Mary chooses a point in the square, what is the probability that it is not in the circle?  *Solution:* The area of the square would be 12 x 12 or 144 units squared. The area of the circle would be 113.04 units squared. The probability that a point is not in the circle would be $\frac{30.96}{144}$ or 21.5% <ins>Example 2:</ins> Jason is tossing a fair coin. He tosses the coin ten times and it lands on heads eight times. If Jason tosses the coin an eleventh time, what is the probability that it will land on heads? *Solution:* The probability would be $\frac{1}{2}$. The result of the eleventh toss does not depend on the previous results. <ins>Example 3:</ins> Devise an experiment using a coin to determine whether a baby is a boy or a girl. Conduct the experiment ten times to determine the gender of ten births. How could a number cube be used to simulate whether a baby is a girl or a boy or girl? <ins>Example 4:</ins> Conduct an experiment using a Styrofoam cup by tossing the cup and recording how it lands. • How many trials were conducted? • How many times did it land right side up? • How many times did it land upside down/ • How many times did it land on its side? • Determine the probability for each of the above results
7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of	**7.SP.8** Students use tree diagrams, frequency tables, and organized lists, and simulations to determine the probability of compound events. <ins>Example 1:</ins> How many ways could the 3 students, Amy, Brenda, and Carla, come in 1st, 2nd and 3rd place?

outcomes in the sample space for which the compound event occurs.

b. Represent for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Solution:
Making an organized list will identify that there are 6 ways for the students to win a race

A, B, C
A, C, B
B, C, A
B, A, C
C, A, B
C, B, A

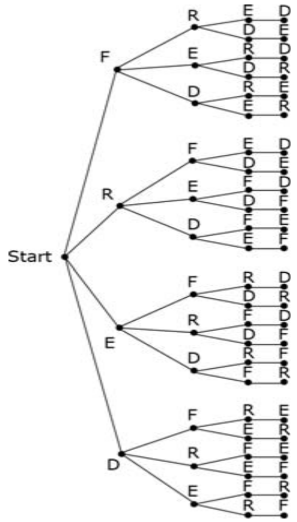
Example 2:
Students conduct a bag pull experiment. A bag contains 5 marbles. There is one red marble, two blue marbles and two purple marbles. Students will draw one marble without replacement and then draw another. What is the sample space for this situation? Explain how the sample space was determined and how it is used to find the probability of drawing one blue marble followed by another blue marble.

Example 3:
A fair coin will be tossed three times. What is the probability that two heads and one tail in any order will results? (Adapted from SREB publication *Getting Students Ready for Algebra I: What Middle Grades Students Need to Know and Be Able to Do*

Solution:

HHT, HTH and THH so the probability would be $\frac{3}{8}$.

Example 4:
Show all possible arrangements of the letters in the word FRED using a tree diagram. If each of the letters is on a tile and drawn at random, what is the probability of drawing the letters F-R-E-D in that order? What is the probability that a “word” will have an F as the first letter?



Solution:
There are 24 possible arrangements (4 choices • 3 choices • 2 choices • 1 choice)

The probability of drawing F-R-E-D in that order is $\frac{1}{24}$.

The probability that a “word” will have an F as the first letter is $\frac{6}{24}$ or $\frac{1}{4}$.

We would like to acknowledge the Arizona Department of Education for allowing us to use some of their examples and graphics.

At A Glance

This page was added to give a snapshot of the mathematical concepts that are new or have been removed from this grade level as well as instructional considerations for the first year of implementation.

New to 8th Grade:

- Integer exponents with numerical bases (8.EE.1)
- Scientific notation, including multiplication and division (8.EE.3 and 8.EE.4)
- Unit rate as slope (8.EE.5)
- Qualitative graphing (8.F.5)
- Transformations (8.G.1 and 8.G.3)
- Congruent and similar figures (characterized through transformations) (8.G.2 and 8.G.4)
- Angles (exterior angles, parallel cut by transversal, angle-angle criterion) (8.G.5)
- Finding diagonal distances on a coordinate plane using the Pythagorean Theorem (8.G.8)
- Volume of cones, cylinders and spheres (8.G.9)
- Two-way tables (8.SP.4)

Moved from 8th Grade:

- Indirect measurement (embedded throughout)
- Linear inequalities (moved to high school)
- Effect of dimension changes (moved to high school)
- Misuses of data (embedded throughout)
- Function notation (moved to high school)
- Point-slope form (moved to high school) and standard form of a linear equation (not in CCSS)

Notes:

- Topics may appear to be similar between the CCSS and the 2003 NCSCOS; however, the CCSS may be presented at a higher cognitive demand.
- **For more detailed information, see the crosswalks (http://www.ncpublicschools.org/acre/standards/common-core-tools)**

Instructional considerations for CCSS implementation in 2012 – 2013:

- Solving proportions with tables, graphs, equations (7.RP.2a, 7.RP.2b, 7.RP.2c, 7.RP.2d) – prerequisite to 8.EE.5
- Identifying the conditions for lengths to make a triangle (7.G.2)
- Supplementary, complementary, vertical and adjacent angles (7.G.5) – prerequisite to 8.G.5
- Finding vertical and horizontal distances on the coordinate plane (6.NS.3) – foundational to 8.G.8
- Mean Absolute Deviation (6.SP.5c) – foundational to standard deviation in Math One standards so could be addressed at that time.

Standards for Mathematical Practice

The Common Core State Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that students complete.

Standards for Mathematical Practice	**Explanations and Examples**
1. Make sense of problems and persevere in solving them.	In grade 8, students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”
2. Reason abstractly and quantitatively.	In grade 8, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others.	In grade 8, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?” “Does that always work?” They explain their thinking to others and respond to others’ thinking.
4. Model with mathematics.	In grade 8, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students solve systems of linear equations and compare properties of functions provided in different forms. Students use scatterplots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 8 may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal.
6. Attend to precision.	In grade 8, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays.
7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. In grade 8, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.

Standards for Mathematical Practice	**Explanations and Examples**
8. Look for and express regularity in repeated reasoning.	In grade 8, students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine more precise rational approximations for irrational numbers. They analyze patterns of repeating decimals to identify the corresponding fraction. During multiple opportunities to solve and model problems, they notice that the slope of a line and rate of change are the same value. Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities.

Grade 8 Critical Areas (from CCSS pg. 52)

The Critical Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction. The Critical Areas for eighth grade can be found on page 52 in the *Common Core State Standards for Mathematics.*

1. **Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations**
   Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

   Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

2. **Grasping the concept of a function and using functions to describe quantitative relationships**
   Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

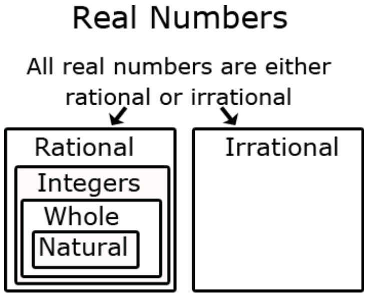
3. **Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem**
   Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

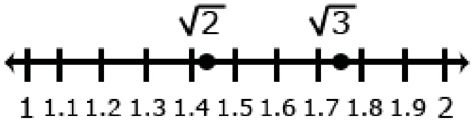
The Number System 8.NS

Common Core Cluster

Know that there are numbers that are not rational, and approximate them by rational numbers.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **Real Numbers, Irrational numbers, Rational numbers, Integers, Whole numbers, Natural numbers, radical, radicand, square roots, perfect squares, cube roots, terminating decimals, repeating decimals, truncate**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	**8.NS.1** Students understand that Real numbers are either rational or irrational. They distinguish between rational and irrational numbers, recognizing that any number that can be expressed as a fraction is a rational number. The diagram below illustrates the relationship between the subgroups of the real number system.  Students recognize that the decimal equivalent of a fraction will either terminate or repeat. Fractions that terminate will have denominators containing only prime factors of 2 and/or 5. This understanding builds on work in 7th grade when students used long division to distinguish between repeating and terminating decimals. Students convert repeating decimals into their fraction equivalent using patterns or algebraic reasoning. One method to find the fraction equivalent to a repeating decimal is shown below. <u>Example 1:</u> Change $0.\overline{4}$ to a fraction. • Let $x = 0.444444$….. • Multiply both sides so that the repeating digits will be in front of the decimal. In this example, one digit repeats so both sides are multiplied by 10, giving $10x = 4.4444444$….

	- Subtract the original equation from the new equation. $$\begin{array}{r} 10x = 4.4444444\ldots. \\ \underline{-x = 0.444444\ldots..} \\ 9x = 4 \end{array}$$ - Solve the equation to determine the equivalent fraction. $$\frac{9x}{9} = \frac{4}{9}$$ $$x = \frac{4}{9}$$ Additionally, students can investigate repeating patterns that occur when fractions have denominators of 9, 99, or 11. <u>Example 2:</u> $\frac{4}{9}$ is equivalent to $0.\overline{4}$, $\frac{5}{9}$ is equivalent to $0.\overline{5}$, etc.
8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*	**8.NS.2** Students locate rational and irrational numbers on the number line. Students compare and order rational and irrational numbers. Students also recognize that square roots may be negative and written as $-\sqrt{28}$. <u>Example 1:</u> Compare $\sqrt{2}$ and $\sqrt{3}$  *Solution:* Statements for the comparison could include: $\sqrt{2}$ and $\sqrt{3}$ are between the whole numbers 1 and 2 $\sqrt{3}$ is between 1.7 and 1.8 $\sqrt{2}$ is less than $\sqrt{3}$ Additionally, students understand that the value of a square root can be approximated between integers and that non-perfect square roots are irrational. <u>Example 2:</u> Find an approximation of $\sqrt{28}$ • Determine the perfect squares $\sqrt{28}$ is between, which would be 25 and 36. • The square roots of 25 and 36 are 5 and 6 respectively, so we know that $\sqrt{28}$ is between 5 and 6. • Since 28 is closer to 25, an estimate of the square root would be closer to 5. One method to get an estimate is to divide 3 (the distance between 25 and 28) by 11 (the distance between the perfect squares of 25 and 36) to get 0.27. • The estimate of $\sqrt{28}$ would be 5.27 (the actual is 5.29).

Expressions and Equations 8.EE

Common Core Cluster

Work with radicals and integer exponents.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **laws of exponents, power, perfect squares, perfect cubes, root, square root, cube root, scientific notation, standard form of a number.** Students should also be able to read and use the symbol: ±

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.	**8.EE.1** In 6th grade, students wrote and evaluated simple numerical expressions with whole number exponents (ie. $5^3 = 5 \bullet 5 \bullet 5 = 125$). Integer (positive and negative) exponents are further developed to generate equivalent numerical expressions when multiplying, dividing or raising a power to a power. Using numerical bases and the laws of exponents, students generate equivalent expressions. Students understand: • Bases must be the same before exponents can be added, subtracted or multiplied. (Example 1) • Exponents are subtracted when like bases are being divided (Example 2) • A number raised to the zero (0) power is equal to one. (Example 3) • Negative exponents occur when there are more factors in the denominator. These exponents can be expressed as a positive if left in the denominator. (Example 4) • Exponents are added when like bases are being multiplied (Example 5) • Exponents are multiplied when an exponents is raised to an exponent (Example 6) • Several properties may be used to simplify an expression (Example 7) Example 1: $\frac{2^3}{5^2} = \frac{8}{25}$ Example 2: $\frac{2^2}{2^6} = 2^{2-6} = 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$ Example 3: $6^0 = 1$ Students understand this relationship from examples such as $\frac{6^2}{6^2}$. This expression could be simplified as $\frac{36}{36} = 1$. Using the laws of exponents this expression could also be written as $6^{2-2} = 6^0$. Combining these gives $6^0 = 1$.

	Example 4: $\frac{3^{-2}}{2^4} = 3^{-2} \text{ x } \frac{1}{2^4} = \frac{1}{3^2} \text{ x } \frac{1}{2^4} = \frac{1}{9} \text{ x } \frac{1}{16} = \frac{1}{144}$ Example 5: $(3^2)(3^4) = (3^{2+4}) = 3^6 = 729$ Example 6: $(4^3)^2 = 4^{3x2} = 4^6 = 4{,}096$ Example 7: $\frac{(3^2)^4}{(3^2)(3^3)} = \frac{3^{2x4}}{3^{2+3}} = \frac{3^8}{3^5} = 3^{8-5} = 3^3 = 27$
8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where *p* is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.	**8.EE.2** Students recognize perfect squares and cubes, understanding that non-perfect squares and non-perfect cubes are irrational. Students recognize that squaring a number and taking the square root √ of a number are inverse operations; likewise, cubing a number and taking the cube root $\sqrt[3]{\ }$ are inverse operations. Example 1: $4^2 = 16$ and $\sqrt{16} = \pm 4$ NOTE: $(-4)^2 = 16$ while $-4^2 = -16$ since the negative is not being squared. This difference is often problematic for students, especially with calculator use. Example 2: $\left(\frac{1}{3}\right)^3 = \left(\frac{1^3}{3^3}\right) = \frac{1}{27}$ and $\sqrt[3]{\frac{1}{27}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}} = \frac{1}{3}$ NOTE: there is no negative cube root since multiplying 3 negatives would give a negative. This understanding is used to solve equations containing square or cube numbers. Rational numbers would have perfect squares or perfect cubes for the numerator and denominator. In the standard, the value of *p* for square root and cube root equations must be positive. Example 3: Solve: $x^2 = 25$ *Solution:* $\sqrt{x^2} = \pm\sqrt{25}$ $x = \pm 5$ NOTE: There are two solutions because 5 • 5 and -5 • -5 will both equal 25.

	Example 4: Solve: $x^2 = \frac{4}{9}$ *Solution:* $\sqrt{x^2} = \pm\sqrt{\frac{4}{9}}$ $x = \pm\frac{2}{3}$ Example 5: Solve: $x^3 = 27$ *Solution:* $\sqrt[3]{x} = \sqrt[3]{27}$ $x = 3$ Example 6: Solve: $x^3 = \frac{1}{8}$ *Solution:* $\sqrt[3]{x} = \sqrt[3]{\frac{1}{8}}$ $x = \frac{1}{2}$ Students understand that in geometry the square root of the area is the length of the side of a square and a cube root of the volume is the length of the side of a cube. Students use this information to solve problems, such as finding the perimeter. Example 7: What is the side length of a square with an area of 49 ft^2? *Solution:* $\sqrt{49}$ = 7 ft. The length of one side is 7 ft.
8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*	**8.EE.3** Students use scientific notation to express very large or very small numbers. Students compare and interpret scientific notation quantities in the context of the situation, recognizing that if the exponent increases by one, the value increases 10 times. Likewise, if the exponent decreases by one, the value decreases 10 times. Students solve problems using addition, subtraction or multiplication, expressing the answer in scientific notation. Example 1: Write 75,000,000,000 in scientific notation. *Solution:* $7.5 \text{ x } 10^{10}$ Example 2: Write 0.0000429 in scientific notation. *Solution:* $4.29 \text{ x } 10^{-5}$

	Example 3: Express 2.45×10^5 in standard form. *Solution:* 245,000 Example 4: How much larger is 6×10^5 compared to 2×10^3 *Solution:* 300 times larger since 6 is 3 times larger than 2 and 10^5 is 100 times larger than 10^3. Example 5: Which is the larger value: 2×10^6 or 9×10^5? *Solution:* 2×10^6 because the exponent is larger
8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	**8.EE.4** Students understand scientific notation as generated on various calculators or other technology. Students enter scientific notation using E or EE (scientific notation), * (multiplication), and ^ (exponent) symbols. Example 1: 2.45E+23 is 2.45×10^{23} and 3.5E-4 is 3.5×10^{-4} (NOTE: There are other notations for scientific notation depending on the calculator being used.) Students add and subtract with scientific notation. Example 2: In July 2010 there were approximately 500 million facebook users. In July 2011 there were approximately 750 million facebook users. How many more users were there in 2011. Write your answer in scientific notation. *Solution:* Subtract the two numbers: $750{,}000{,}000 - 500{,}000{,}000 = 250{,}000{,}000 \rightarrow 2.5 \times 10^8$ Students use laws of exponents to multiply or divide numbers written in scientific notation, writing the product or quotient in proper scientific notation. Example 3: $(6.45 \times 10^{11})(3.2 \times 10^4) = (6.45 \times 3.2)(10^{11} \times 10^4)$ *Rearrange factors* $= 20.64 \times 10^{15}$ *Add exponents when multiplying powers of 10* $= 2.064 \times 10^{16}$ *Write in scientific notation* Example 4: $\frac{3.45 \times 10^5}{6.7 \times 10^{-2}} \quad \frac{6.3}{1.6} \times 10^{5-(-2)}$ *Subtract exponents when dividing powers of 10* $= 0.515 \times 10^7$ *Write in scientific notation* $= 5.15 \times 10^6$ Example 5: $(0.0025)(5.2 \times 10^4) = (2.5 \times 10^{-3})(5.2 \times 10^5)$ *Write factors in scientific notation* $= (2.5 \times 5.2)(10^{-3} \times 10^5)$ *Rearrange factors* $= 13 \times 10^2$ *Add exponents when multiplying powers of 10* $= 1.3 \times 10^3$ *Write in scientific notation*

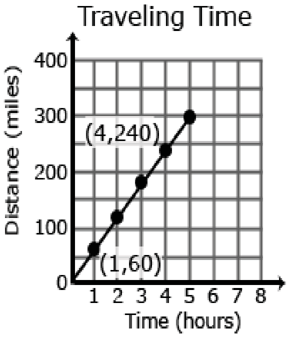
	Example 6: The speed of light is 3×10^{8} meters/second. If the sun is 1.5×10^{11} meters from earth, how many seconds does it take light to reach the earth? Express your answer in scientific notation. *Solution*: 5×10^{2} (light)(x) = sun, where x is the time in seconds $(3 \times 10^{8})x = 1.5 \times 10^{11}$ $\frac{1.5 \times 10^{11}}{3 \times 10^{8}}$ Students understand the magnitude of the number being expressed in scientific notation and choose an appropriate corresponding unit. Example 7: 3×10^{8} is equivalent to 300 million, which represents a large quantity. Therefore, this value will affect the unit chosen.

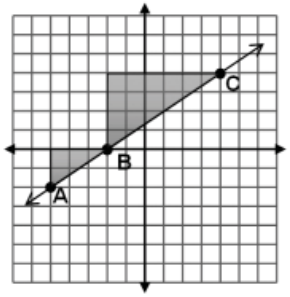
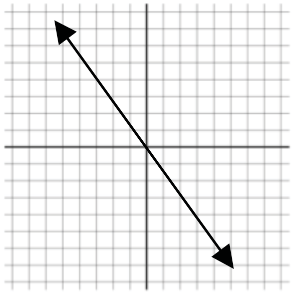
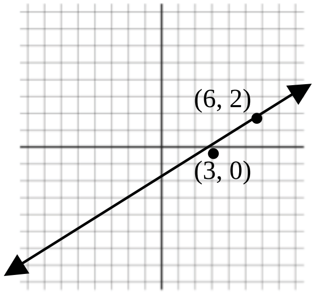
Expressions and Equations 8.EE

Common Core Cluster

Understand the connections between proportional relationships, lines, and linear equations.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **unit rate, proportional relationships, slope, vertical, horizontal, similar triangles, *y*-intercept**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*	**8.EE.5** Students build on their work with unit rates from 6th grade and proportional relationships in 7th grade to compare graphs, tables and equations of proportional relationships. Students identify the unit rate (or slope) in graphs, tables and equations to compare two proportional relationships represented in different ways. Example 1: Compare the scenarios to determine which represents a greater speed. Explain your choice including a written description of each scenario. Be sure to include the unit rates in your explanation. **Scenario 1:**  **Scenario 2:** $y = 55x$ x is time in hours y is distance in miles *Solution:* Scenario 1 has the greater speed since the unit rate is 60 miles per hour. The graph shows this rate since 60 is the distance traveled in one hour. Scenario 2 has a unit rate of 55 miles per hour shown as the coefficient in the equation. Given an equation of a proportional relationship, students draw a graph of the relationship. Students recognize that the unit rate is the coefficient of *x* and that this value is also the slope of the line.
8.EE.6 Use similar triangles to explain why the slope *m* is the same between any two distinct points on a	**8.EE.6** Triangles are similar when there is a constant rate of proportionality between them. Using a graph, students construct triangles between two points on a line and compare the sides to understand that the slope (ratio of rise to run) is the same between any two points on a line.

non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	<u>Example 1:</u> The triangle between A and B has a vertical height of 2 and a horizontal length of 3. The triangle between B and C has a vertical height of 4 and a horizontal length of 6. The simplified ratio of the vertical height to the horizontal length of both triangles is 2 to 3, which also represents a slope of $\frac{2}{3}$ for the line, indicating that the triangles are similar.  Given an equation in slope-intercept form, students graph the line represented. Students write equations in the form $y = mx$ for lines going through the origin, recognizing that m represents the slope of the line. <u>Example 2:</u> Write an equation to represent the graph to the right.  *Solution:* $y = -\frac{3}{2}x$ Students write equations in the form $y = mx + b$ for lines not passing through the origin, recognizing that m represents the slope and b represents the y-intercept.  *Solution:* $y = \frac{2}{3}x - 2$

Expressions and Equations 8.EE

Common Core Cluster

Analyze and solve linear equations and pairs of simultaneous linear equations.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **intersecting, parallel lines, coefficient, distributive property, like terms, substitution, system of linear equations**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
8.EE.7 Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, *or* $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	**8.EE.7** Students solve one-variable equations including those with the variables being on both sides of the equals sign. Students recognize that the solution to the equation is the value(s) of the variable, which make a true equality when substituted back into the equation. Equations shall include rational numbers, distributive property and combining like terms. <u>Example 1:</u> Equations have one solution when the variables do not cancel out. For example, $10x – 23 = 29 – 3x$ can be solved to $x = 4$. This means that when the value of x is 4, both sides will be equal. If each side of the equation were treated as a linear equation and graphed, the solution of the equation represents the coordinates of the point where the two lines would intersect. In this example, the ordered pair would be (4, 17). $10 \bullet 4 – 23 = 29 – 3 \bullet 4$ $40 – 23 = 29 – 12$ $17 = 17$ <u>Example 2:</u> Equations having no solution have variables that will cancel out and constants that are not equal. This means that there is not a value that can be substituted for x that will make the sides equal. $-x + 7 – 6x = 19 – 7x$ *Combine like terms* $-7x + 7 = 19 – 7x$ *Add 7x to each side* $7 \neq 19$ This solution means that no matter what value is substituted for x the final result will never be equal to each other. If each side of the equation were treated as a linear equation and graphed, the lines would be parallel.

	<u>Example 3:</u> An equation with infinitely many solutions occurs when both sides of the equation are the same. Any value of x will produce a valid equation. For example the following equation, when simplified will give the same values on both sides. $-\frac{1}{2}(36a - 6) = \frac{3}{4}(4 - 24a)$ $-18a + 3 = 3 - 18a$ If each side of the equation were treated as a linear equation and graphed, the graph would be the same line. Students write equations from verbal descriptions and solve. <u>Example 4:</u> Two more than a certain number is 15 less than twice the number. Find the number. *Solution:* $n + 2 = 2n - 15$ $17 = n$
8.EE.8 Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.	**8.EE.8** Systems of linear equations can also have one solution, infinitely many solutions or no solutions. Students will discover these cases as they graph systems of linear equations and solve them algebraically. Students graph a system of two linear equations, recognizing that the ordered pair for the point of intersection is the x-value that will generate the given y-value for both equations. Students recognize that graphed lines with one point of intersection (different slopes) will have one solution, parallel lines (same slope, different y-intercepts) have no solutions, and lines that are the same (same slope, same y-intercept) will have infinitely many solutions. By making connections between algebraic and graphical solutions and the context of the system of linear equations, students are able to make sense of their solutions. Students need opportunities to work with equations and context that include whole number and/or decimals/fractions. Students define variables and create a system of linear equations in two variables <u>Example 1:</u> 1. Plant A and Plant B are on different watering schedules. This affects their rate of growth. Compare the growth of the two plants to determine when their heights will be the same. *Solution:* Let W = number of weeks Let H = height of the plant after W weeks
b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.*	
c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For*	

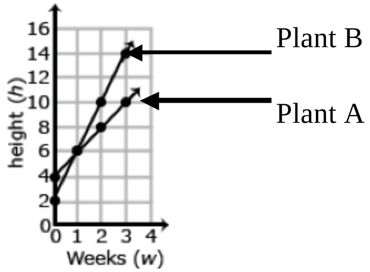
example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Plant A		
W	H	
0	4	(0, 4)
1	6	(1, 6)
2	8	(2, 8)
3	10	(3, 10)

Plant B		
W	H	
0	2	(0, 2)
1	6	(1, 6)
2	10	(2, 10)
3	14	(3, 14)

2. Based on the coordinates from the table, graph lines to represent each plant.

Solution:



3. Write an equation that represents the growth rate of Plant A and Plant B.

Solution: Plant A $H = 2W + 4$
Plant B $H = 4W + 2$

4. At which week will the plants have the same height?

Solution:

$2W + 4 = 4W + 2$ *Set height of Plant A equal to height of Plant B*
$2W – 2W + 4 = 4W – 2W + 2$ *Solve for W*
$4 = 2W + 2$
$4 – 2 = 2W + 2 – 2$
$\frac{2}{2} = \frac{2W}{2}$
$1 = W$

After one week, the height of Plant A and Plant B are both 6 inches.

Check: $2(1) + 4 = 4(1) + 2$
$2 + 4 = 4 + 2$
$6 = 6$

Given two equations in slope-intercept form (Example 1) or one equation in standard form and one equation in slope-intercept form, students use substitution to solve the system.

Example 2:
Solve: Victor is half as old as Maria. The sum of their ages is 54. How old is Victor?

Solution: Let v = Victor's age
Let m = Maria's age

$$\begin{cases} v + m = 54 \\ v = \frac{1}{2} m \end{cases}$$

$\frac{1}{2} m + m = 54$ *Substitute ½m for v in the first equation*

$1\frac{1}{2} m = 54$

$m = 36$

If Maria is 36, then substitute 36 into $v + m = 54$ to find Victor's age of 18.

Note: Students are not expected to change linear equations written in standard form to slope-intercept form or solve systems using elimination.

For many real world contexts, equations may be written in standard form. Students are not expected to change the standard form to slope-intercept form. However, students may generate ordered pairs recognizing that the values of the ordered pairs would be solutions for the equation. For example, in the equation above, students could make a list of the possible ages of Victor and Maria that would add to 54. The graph of these ordered pairs would be a line with all the possible ages for Victor and Maria.

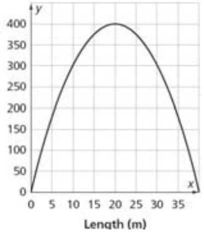
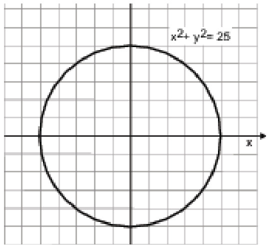
Victor	**Maria**
20	34
10	44
50	4
29	25

Functions 8.F

Common Core Cluster

Define, evaluate, and compare functions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **functions, *y*-value, *x*-value, vertical line test, input, output, rate of change, linear function, non-linear function**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1] [1] Function notation is not required in Grade 8.	**8.F.1** Students understand rules that take x as input and gives y as output is a function. Functions occur when there is exactly one y-value is associated with any x-value. Using y to represent the output we can represent this function with the equations $y = x^2 + 5x + 4$. Students are **not** expected to use the function notation *f(x)* at this level. Students identify functions from equations, graphs, and tables/ordered pairs. **Graphs** Students recognize graphs such as the one below is a function using the vertical line test, showing that each x-value has only one y-value;  whereas, graphs such as the following are not functions since there are 2 y-values for multiple x-value. 

Tables or Ordered Pairs
Students read tables or look at a set of ordered pairs to determine functions and identify equations where there is only one output (*y*-value) for each input (*x*-value).

Functions

x	**y**
0	3
1	9
2	27

Not A Function

x	**y**
16	4
16	-4
25	5
25	-5

{(0, 2), (1, 3), (2, 5), (3, 6)}

Equations
Students recognize equations such as $y = x$ or $y = x^2 + 3x + 4$ as functions; whereas, equations such as $x^2 + y^2 = 25$ are not functions.

8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

8.F.2 Students compare two functions from different representations.

Example 1:
Compare the following functions to determine which has the greater rate of change.
Function 1: $y = 2x + 4$
Function 2:

x	**y**
-1	-6
0	-3
2	3

Solution: The rate of change for function 1 is 2; the rate of change for function 2 is 3. Function 2 has the greater rate of change.

Example 2:
Compare the two linear functions listed below and determine which has a negative slope.

Function 1: Gift Card
Samantha starts with $20 on a gift card for the bookstore. She spends $3.50 per week to buy a magazine. Let y be the amount remaining as a function of the number of weeks, *x*.

x	y
0	20
1	16.50
2	13.00
3	9.50

Function 2: Calculator rental
The school bookstore rents graphing calculators for $5 per month. It also collects a non-refundable fee of $10.00 for the school year. Write the rule for the total cost (c) of renting a calculator as a function of the number of months (*m*).
$c = 10 + 5m$

Solution: Function 1 is an example of a function whose graph has a negative slope. Both functions have a positive starting amount; however, in function 1, the amount decreases 3.50 each week, while in function 2, the amount increases 5.00 each month.

NOTE: Functions could be expressed in standard form. However, the intent is not to change from standard form to slope-intercept form but to use the standard form to generate ordered pairs. Substituting a zero (0) for *x* and *y* will generate two ordered pairs. From these ordered pairs, the slope could be determined.
Example 3:
$2x + 3y = 6$

Let $x = 0$: $2(0) + 3y = 6$
$3y = 6$
$\frac{3y}{3} = \frac{6}{3}$
$y = 2$

Ordered pair: (0, 2)

Let $y = 0$: $2x + 3(0) = 6$
$2x = 6$
$\frac{2x}{2} = \frac{6}{2}$
$x = 3$

Ordered pair: (3, 0)

Using (0, 2) and (3, 0) students could find the slope and make comparisons with another function.

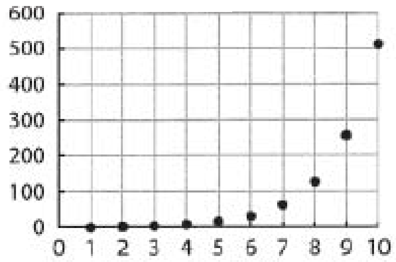
8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

8.F.3 Students understand that linear functions have a constant rate of change between any two points. Students use equations, graphs and tables to categorize functions as linear or non-linear.

Example 1:
Determine if the functions listed below are linear or non-linear. Explain your reasoning.
1. $y = -2x^2 + 3$
2. $y = 0.25 + 0.5(x - 2)$
3. $A = \pi r^2$
4.

X	Y
1	12
2	7
3	4
4	3
5	4
6	7

5.



	Solution: 1. Non-linear 2. Linear 3. Non-linear 4. Non-linear; there is not a constant rate of change 5. Non-linear; the graph curves indicating the rate of change is not constant.

Functions 8.F

Common Core Cluster

Use functions to model relationships between quantities.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **linear relationship, rate of change, slope, initial value, y-intercept**

Common Core Standard	Unpacking What does this standard mean that a student will know and be able to do?

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.4 Students identify the rate of change (slope) and initial value (y-intercept) from tables, graphs, equations or verbal descriptions to write a function (linear equation). Students understand that the equation represents the relationship between the x-value and the y-value; what math operations are performed with the x-value to give the y-value. Slopes could be undefined slopes or zero slopes.

Tables:

Students recognize that in a table the y-intercept is the y-value when x is equal to 0. The slope can be determined by finding the ratio $\frac{y}{x}$ between the change in two y-values and the change between the two corresponding x-values.

Example 1:

Write an equation that models the linear relationship in the table below.

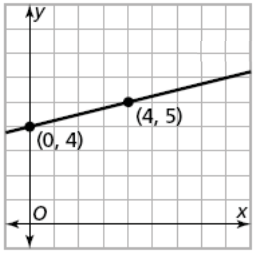
x	y
-2	8
0	2
1	-1

Solution: The y-intercept in the table below would be (0, 2). The distance between 8 and -1 is 9 in a negative direction → -9; the distance between -2 and 1 is 3 in a positive direction. The slope is the ratio of rise to run or $\frac{y}{x}$ or $\frac{-9}{3}$ = -3. The equation would be $y = -3x + 2$

Graphs:

Using graphs, students identify the y-intercept as the point where the line crosses the y-axis and the slope as the rise.
run

Example 2:
Write an equation that models the linear relationship in the graph below.



Solution: The y-intercept is 4. The slope is ¼ , found by moving up 1 and right 4 going from (0, 4) to (4, 5). The linear equation would be $y = \frac{1}{4} x + 4$.

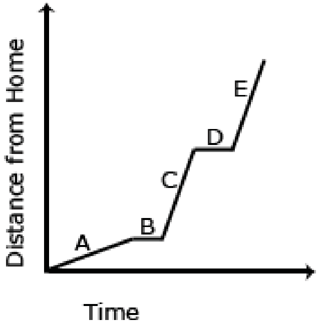
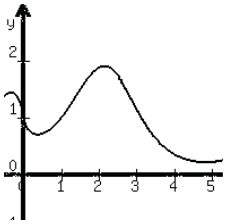
Equations:
In a linear equation the coefficient of x is the slope and the constant is the y-intercept. Students need to be given the equations in formats other than $y = mx + b$, such as $y = ax + b$ (format from graphing calculator), $y = b + mx$ (often the format from contextual situations), etc.

Point and Slope:
Students write equations to model lines that pass through a given point with the given slope.
Example 2:
A line has a zero slope and passes through the point (-5, 4). What is the equation of the line?
Solution: $y = 4$

Example 3:
Write an equation for the line that has a slope of ½ and passes though the point (-2, 5)
Solution: $y = \frac{1}{2} x + 6$
Students could multiply the slope ½ by the x-coordinate -2 to get -1. Six (6) would need to be added to get to 5, which gives the linear equation.

Students also write equations given two ordered pairs. **Note that point-slope form is not an expectation at this level.** Students use the slope and y-intercepts to write a linear function in the form $y = mx + b$.

Contextual Situations:
In contextual situations, the y-intercept is generally the starting value or the value in the situation when the independent variable is 0. The slope is the rate of change that occurs in the problem. Rates of change can often occur over years. In these situations it is helpful for the years to be “converted” to 0, 1, 2, etc. For example, the years of 1960, 1970, and 1980 could be represented as 0 (for 1960), 10 (for 1970) and 20 (for 1980).

	<u>Example 4:</u> The company charges $45 a day for the car as well as charging a one-time $25 fee for the car’s navigation system (GPS). Write an expression for the cost in dollars, *c*, as a function of the number of days, *d*, the car was rented. *Solution:* $C = 45d + 25$ Students interpret the rate of change and the *y*-intercept in the context of the problem. In Example 3, the rate of change is 45 (the cost of renting the car) and that initial cost (the first day charge) also includes paying for the navigation system. Classroom discussion about one-time fees vs. recurrent fees will help students model contextual situations.
8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	**8.F.5** Given a verbal description of a situation, students sketch a graph to model that situation. Given a graph of a situation, students provide a verbal description of the situation. <u>Example 1:</u> The graph below shows a John’s trip to school. He walks to his Sam’s house and, together, they ride a bus to school. The bus stops once before arriving at school. Describe how each part A – E of the graph relates to the story. *Solution:* A John is walking to Sam’s house at a constant rate. B John gets to Sam’s house and is waiting for the bus. C John and Sam are riding the bus to school. The bus is moving at a constant rate, faster than John’s walking rate. D The bus stops. E The bus resumes at the same rate as in part C.  <u>Example 2:</u> Describe the graph of the function between $x = 2$ and $x = 5$?  *Solution:* The graph is non-linear and decreasing.

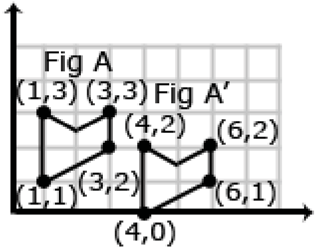
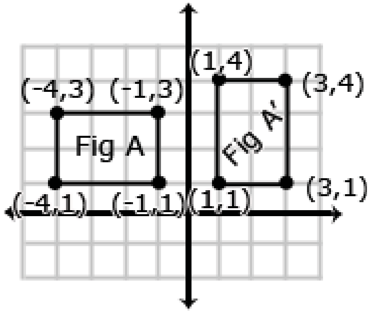
Geometry 8.G

Common Core Cluster

Understand congruence and similarity using physical models, transparencies, or geometry software.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **translations, rotations, reflections, line of reflection, center of rotation, clockwise, counterclockwise, parallel lines, betweenness, congruence, $\cong$, reading A' as "A prime", similarity, dilations, pre-image, image, rigid transformations, exterior angles, interior angles, alternate interior angles, angle-angle criterion, deductive reasoning, vertical angles, adjacent, supplementary, complementary, corresponding, scale factor, transversal, parallel**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
8.G.1 Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	**8.G.1** Students use compasses, protractors and rulers or technology to explore figures created from translations, reflections and rotations. Characteristics of figures, such as lengths of line segments, angle measures and parallel lines, are explored before the transformation (pre-image) and after the transformation (image). Students understand that these transformations produce images of exactly the same size and shape as the pre-image and are known as rigid transformations.

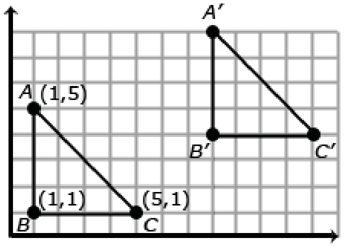
8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	**8.G.2** This standard is the students' introduction to congruency. Congruent figures have the same shape and size. Translations, reflections and rotations are examples of rigid transformations. A rigid transformation is one in which the pre-image and the image both have exactly the same size and shape since the measures of the corresponding angles and corresponding line segments remain equal (are congruent). Students examine two figures to determine congruency by identifying the rigid transformation(s) that produced the figures. Students recognize the symbol for congruency (≅) and write statements of congruency. <u>Example 1:</u> Is Figure A congruent to Figure A'? Explain how you know.  *Solution:* These figures are congruent since A' was produced by translating each vertex of Figure A 3 to the right and 1 down <u>Example 2:</u> Describe the sequence of transformations that results in the transformation of Figure A to Figure A'.  *Solution:* Figure A' was produced by a 90º clockwise rotation around the origin.

8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

8.G.3 Students identify resulting coordinates from translations, reflections, and rotations (90º, 180º and 270º both clockwise and counterclockwise), recognizing the relationship between the coordinates and the transformation.

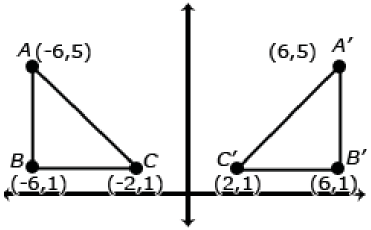
Translations

Translations move the object so that every point of the object moves in the same direction as well as the same distance. In a translation, the translated object is *congruent* to its pre-image. Triangle ABC has been translated 7 units to the right and 3 units up. To get from A (1,5) to A' (8,8), move A 7 units to the right (from $x = 1$ to $x = 8$) and 3 units up (from $y = 5$ to $y = 8$). Points B and C also move in the same direction (7 units to the right and 3 units up), resulting in the same changes to each coordinate.



Reflections

A reflection is the "flipping" of an object over a line, known as the "line of reflection". In the 8th grade, the line of reflection will be the *x*-axis and the *y*-axis. Students recognize that when an object is reflected across the *y*-axis, the reflected *x*-coordinate is the opposite of the pre-image x-coordinate (see figure below).

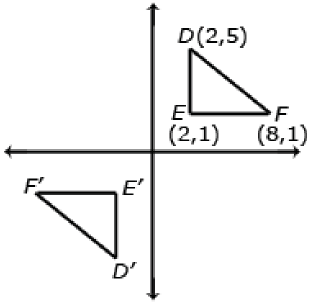


Likewise, a reflection across the *x*-axis would change a pre-image coordinate (3, -8) to the image coordinate of (3, 8) -- note that the reflected *y*-coordinate is opposite of the pre-image *y*-coordinate.

Rotations

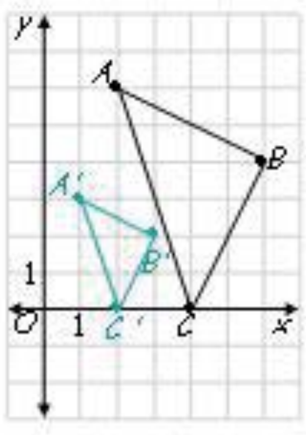
A rotation is a transformation performed by "spinning" the figure around a fixed point known as the center of rotation. The figure may be rotated clockwise or counterclockwise up to 360º (at 8th grade, rotations will be around the origin and a multiple of 90º). In a rotation, the rotated object is *congruent* to its pre-image

Consider when triangle DEF is 180˚ clockwise about the origin. The coordinate of triangle DEF are D(2,5), E(2,1), and F(8,1). When rotated 180˚ about the origin, the new coordinates are D’(-2,-5), E’(-2,-1) and F’(-8,-1). In this case, each coordinate is the opposite of its pre-image (see figure below).



Dilations

A dilation is a non-rigid transformation that moves each point along a ray which starts from a fixed center, and multiplies distances from this center by a common scale factor. Dilations enlarge (scale factors greater than one) or reduce (scale factors less than one) the size of a figure by the scale factor. In 8th grade, dilations will be from the origin. The dilated figure is *similar* to its pre-image.

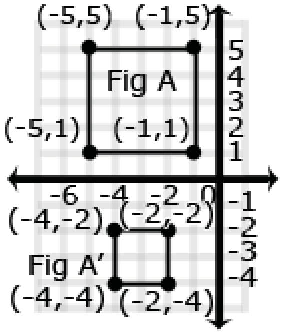
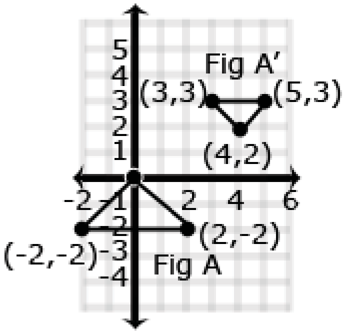


The coordinates of A are (2, 6); A’ (1, 3). The coordinates of B are (6, 4) and B’ are (3, 2). The coordinates of C are (4, 0) and C’ are (2, 0). Each of the image coordinates is ½ the value of the pre-image coordinates indicating a scale factor of ½.

The scale factor would also be evident in the length of the line segments using the ratio: $\frac{\text{image length}}{\text{pre-image length}}$

Students recognize the relationship between the coordinates of the pre-image, the image and the scale factor for a dilation from the origin. Using the coordinates, students are able to identify the scale factor (image/pre-image).

Students identify the transformation based on given coordinates. For example, the pre-image coordinates of a triangle are A(4, 5), B(3, 7), and C(5, 7). The image coordinates are A(-4, 5), B(-3, 7), and C(-5, 7). What transformation occurred?

8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	**8.G.4** Similar figures and similarity are first introduced in the 8th grade. Students understand similar figures have congruent angles and sides that are proportional. Similar figures are produced from dilations. Students describe the sequence that would produce similar figures, including the scale factors. Students understand that a scale factor greater than one will produce an enlargement in the figure, while a scale factor less than one will produce a reduction in size. <u>Example1:</u> Is Figure A similar to Figure A'? Explain how you know.  *Solution*: Dilated with a scale factor of ½ then reflected across the *x*-axis, making Figures A and A' similar. Students need to be able to identify that triangles are similar or congruent based on given information. <u>Example 2:</u> Describe the sequence of transformations that results in the transformation of Figure A to Figure A'.  *Solution*: 90° clockwise rotation, translate 4 right and 2 up, dilation of ½. In this case, the scale factor of the dilation can be found by using the horizontal distances on the triangle (image = 2 units; pre-image = 4 units)

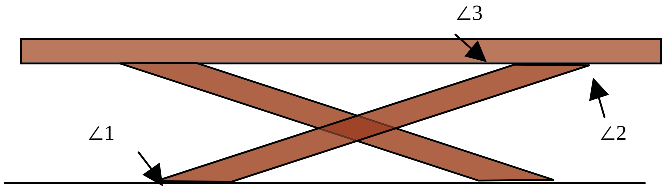
8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

8.G.5 Students use exploration and deductive reasoning to determine relationships that exist between the following: a) angle sums and exterior angle sums of triangles, b) angles created when parallel lines are cut by a transversal, and c) the angle-angle criterion for similarity of triangle.

Students construct various triangles and find the measures of the interior and exterior angles. Students make conjectures about the relationship between the measure of an exterior angle and the other two angles of a triangle. (the measure of an exterior angle of a triangle is equal to the sum of the measures of the other two interior angles) and the sum of the exterior angles (360º). Using these relationships, students use deductive reasoning to find the measure of missing angles.

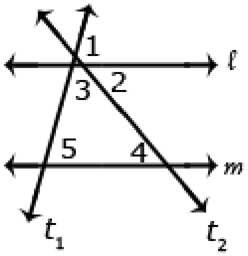
Students construct parallel lines and a transversal to examine the relationships between the created angles. Students recognize vertical angles, adjacent angles and supplementary angles from 7th grade and build on these relationships to identify other pairs of congruent angles. Using these relationships, students use deductive reasoning to find the measure of missing angles.

Example 1:
You are building a bench for a picnic table. The top of the bench will be parallel to the ground. If $m\angle 1 = 148^\circ$, find $m\angle 2$ and $m\angle 3$. Explain your answer.



Solution:
Angle 1 and angle 2 are alternate interior angles, giving angle 2 a measure of 148º. Angle 2 and angle 3 are supplementary. Angle 3 will have a measure of 32º so the $m\angle 2 + m\angle 3 = 180^\circ$

Example 2:
Show that $m\angle 3 + m\angle 4 + m\angle 5 = 180^\circ$ if line l and m are parallel lines and t_1 and t_2 are transversals.



Solution: $\angle 1 + \angle 2 + \angle 3 = 180^\circ$

$\angle 5 \cong \angle 1$ corresponding angles are congruent therefore $\angle 1$ can be substituted for $\angle 5$

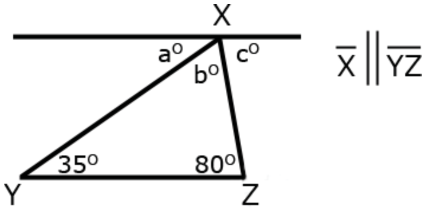
$\angle 4 \cong \angle 2$ alternate interior angles are congruent therefore $\angle 4$ can be substituted for $\angle 2$

Therefore $\angle 3 + \angle 4 + \angle 5 = 180^\circ$

Students can informally conclude that the sum of the angles in a triangle is 180º (the angle-sum theorem) by applying their understanding of lines and alternate interior angles.

Example 3:

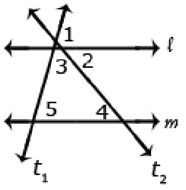
In the figure below Line *X* is parallel to Line $\overline{YZ}$. Prove that the sum of the angles of a triangle is 180º.



Solution: Angle *a* is 35º because it alternates with the angle inside the triangle that measures 35º. Angle *c* is 80º because it alternates with the angle inside the triangle that measures 80º. Because lines have a measure of 180º, and angles *a* + *b* + *c* form a straight line, then angle *b* must be 65 º → 180 – (35 + 80) = 65. Therefore, the sum of the angles of the triangle is 35º + 65 º + 80 º.

Example 4:

What is the measure of angle 5 if the measure of angle 2 is 45º and the measure of angle 3 is 60º?



Solution: Angles 2 and 4 are alternate interior angles, therefore the measure of angle 4 is also 45º. The measure of angles 3, 4 and 5 must add to 180º. If angles 3 and 4 add to 105º the angle 5 must be equal to 75º.

Students construct various triangles having line segments of different lengths but with two corresponding congruent angles. Comparing ratios of sides will produce a constant scale factor, meaning the triangles are similar. Students solve problems with similar triangles.

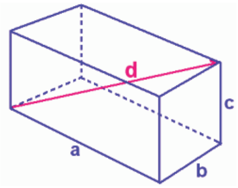
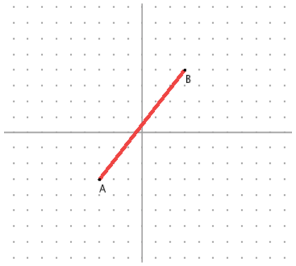
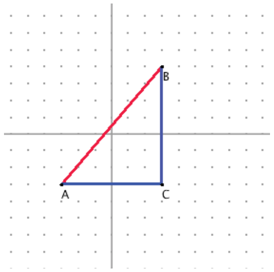
Geometry 8.G

Common Core Cluster

Understand and apply the Pythagorean Theorem.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **right triangle, hypotenuse, legs, Pythagorean Theorem, Pythagorean triple**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
8.G.6 Explain a proof of the Pythagorean Theorem and its converse.	**8.G.6** Using models, students explain the Pythagorean Theorem, understanding that the sum of the squares of the legs is equal to the square of the hypotenuse in a right triangle. Students also understand that given three side lengths with this relationship forms a right triangle. Example 1: The distance from Jonestown to Maryville is 180 miles, the distance from Maryville to Elm City is 300 miles, and the distance from Elm City to Jonestown is 240 miles. Do the three towns form a right triangle? Why or why not? *Solution*: If these three towns form a right triangle, then 300 would be the hypotenuse since it is the greatest distance. $180^2 + 240^2 = 300^2$ $32400 + 57600 = 90000$ $90000 = 90000$ ✓ These three towns form a right triangle.
8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	**8.G.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. Example 1: The Irrational Club wants to build a tree house. They have a 9-foot ladder that must be propped diagonally against the tree. If the base of the ladder is 5 feet from the bottom of the tree, how high will the tree house be off the ground? *Solution*: $a^2 + 5^2 = 9^2$ $a^2 + 25 = 81$ $a^2 = 56$ $\sqrt{a^2} = \sqrt{56}$ $a = \sqrt{56}$ or ~7.5 Example 2: Find the length of d in the figure to the right if a = 8 in., b = 3 in. and c = 4in.

	Solution: First find the distance of the hypotenuse of the triangle formed with legs *a* and *b*.  $8^2 + 3^2 = c^2$ $64^2 + 9^2 = c^2$ $73 = c^2$ $\sqrt{73} = \sqrt{c^2}$ $\sqrt{73}$ in. = c The $\sqrt{73}$ is the length of the base of a triangle with *c* as the other leg and *d* is the hypotenuse. To find the length of *d*: $\sqrt{73}^2 + 4^2 = d^2$ $73 + 16 = d^2$ $89 = d^2$ $\sqrt{89} = \sqrt{d^2}$ $\sqrt{89}$ in. = d Based on this work, students could then find the volume or surface area.
8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	**8.G.8** One application of the Pythagorean Theorem is finding the distance between two points on the coordinate plane. Students build on work from 6th grade (finding vertical and horizontal distances on the coordinate plane) to determine the lengths of the legs of the right triangle drawn connecting the points. Students understand that the line segment between the two points is the length of the hypotenuse. NOTE: The use of the distance formula is not an expectation. <u>Example 1:</u> Find the length of $\overline{AB}$.  *Solution:* 1. Form a right triangle so that the given line segment is the hypotenuse. 2. Use Pythagorean Theorem to find the distance (length) between the two points. $6^2 + 7^2 = c^2$ $36 + 49 = c^2$ $85 = c^2$ 

Example 2:
Find the distance between (-2, 4) and (-5, -6).
Solution:
The distance between -2 and -5 is the horizontal length; the distance between 4 and -6 is the vertical distance.
Horizontal length: 3
Vertical length: 10

$10^2 + 3^2 = c^2$
$100 + 9 = c^2$
$109 = c^2$
$\sqrt{109} = \sqrt{c^2}$
$\sqrt{109} = c$

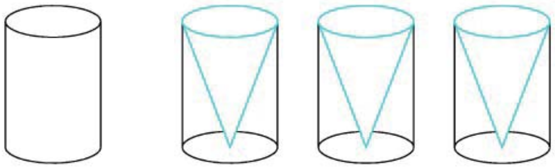
Students find area and perimeter of two-dimensional figures on the coordinate plane, finding the distance between each segment of the figure. (Limit one diagonal line, such as a right trapezoid or parallelogram)

Geometry 8.G

Common Core Cluster

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **cones, cylinders, spheres, radius, volume, height, Pi**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	**8.G.9** Students build on understandings of circles and volume from 7th grade to find the volume of cylinders, finding the area of the base πr^2 and multiplying by the number of layers (the height).  $V = \pi r^2 h$ find the area of the base and multiply by the number of layers Students understand that the volume of a cylinder is 3 times the volume of a cone having the same base area and height or that the volume of a cone is $\frac{1}{3}$ the volume of a cylinder having the same base area and height.  $V = \frac{1}{3}\pi r^2 h$ or $V = \underline{\pi r^2 h}$ A sphere can be enclosed with a cylinder, which has the same radius and height of the sphere (Note: the height of the cylinder is twice the radius of the sphere). If the sphere is flattened, it will fill $\frac{2}{3}$ of the cylinder. Based on this model, students understand that the volume of a sphere is $\frac{2}{3}$ the volume of a cylinder with the same radius and height. The height of the cylinder is the same as the diameter of the sphere or $2r$. Using this information, the formula for the volume of the sphere can be derived in the following way:

$V = \pi r^2 h$ *cylinder volume formula*

$V = \frac{2}{3}\pi r^2 h$ *multiply by* $\frac{2}{3}$ *since the volume of a sphere is* $\frac{2}{3}$ *the cylinder's volume*

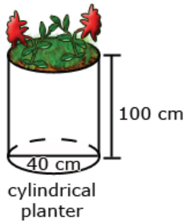
$V = \frac{2}{3}\pi r^2 2r$ *substitute 2r for height since 2r is the height of the sphere*

$V = \frac{4}{3}\pi r^3$ *simplify*

Students find the volume of cylinders, cones and spheres to solve real world and mathematical problems. Answers could also be given in terms of Pi.

Example 1:

James wanted to plant pansies in his new planter. He wondered how much potting soil he should buy to fill it. Use the measurements in the diagram below to determine the planter's volume.



Solution:

$V = \pi r^2 h$

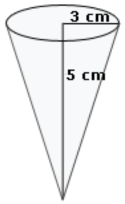
$V = 3.14\ (40)^2(100)$

$V = 502{,}400\ \text{cm}^3$

The answer could also be given in terms of π: $V = 160{,}000\ \pi$

Example 2:

How much yogurt is needed to fill the cone to the right? Express your answers in terms of Pi.



Solution:

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(3^2)(5)$

$V = \frac{1}{3}\pi\ (45)$

$V = 15\ \pi\ \text{cm}^3$

	Example 3: Approximately, how much air would be needed to fill a soccer ball with a radius of 14 cm? *Solution:* $V = \frac{4}{3}\pi r^3$ $V = \frac{4}{3}(3.14)(14^3)$ $V = 11.5\ \text{cm}^3$ “Know the formula” does not mean memorization of the formula. To “know” means to have an understanding of ***why*** the formula works and how the formula relates to the measure (volume) and the figure. This understanding should be for *all* students. **Note:** At this level composite shapes will not be used and only volume will be calculated.

Statistics and Probability 8.SP

Common Core Cluster

Investigate patterns of association in bivariate data.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **bivariate data, scatter plot, linear model, clustering, linear association, non-linear association, outliers, positive association, negative association, categorical data, two-way table, relative frequency**

Common Core Standard

8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

Unpacking

What does this standard mean that a student will know and be able to do?

8.SP.1 Bivariate data refers to two-variable data, one to be graphed on the *x*-axis and the other on the *y*-axis. Students represent numerical data on a scatter plot, to examine relationships between variables. They analyze scatter plots to determine if the relationship is linear (positive, negative association or no association) or non-linear. Students can use tools such as those at the National Center for Educational Statistics to create a graph or generate data sets. (http://nces.ed.gov/nceskids/createagraph/default.aspx)
Data can be expressed in years. In these situations it is helpful for the years to be “converted” to 0, 1, 2, etc. For example, the years of 1960, 1970, and 1980 could be represented as 0 (for 1960), 10 (for 1970) and 20 (for 1980).

<u>Example 1:</u>
Data for 10 students’ Math and Science scores are provided in the chart. Describe the association between the Math and Science scores.

Student	1	2	3	4	5	6	7	8	9	10
Math	64	50	85	34	56	24	72	63	42	93
Science	68	70	83	33	60	27	74	63	40	96

Solution: This data has a positive association.

<u>Example 2:</u>
Data for 10 students’ Math scores and the distance they live from school are provided in the table below. Describe the association between the Math scores and the distance they live from school.

Student	1	2	3	4	5	6	7	8	9	10
Math	64	50	85	34	56	24	72	63	42	93
Distance from School (miles)	0.5	1.8	1	2.3	3.4	0.2	2.5	1.6	0.8	2.5

Solution: There is no association between the math score and the distance a student lives from school.

Example 3:

Data from a local fast food restaurant is provided showing the number of staff members and the average time for filling an order are provided in the table below. Describe the association between the number of staff and the average time for filling an order.

Number of Staff	3	4	5	6	7	8
Average time to fill order (seconds)	56	24	72	63	42	93

Solution: There is a positive association.

Example 4:

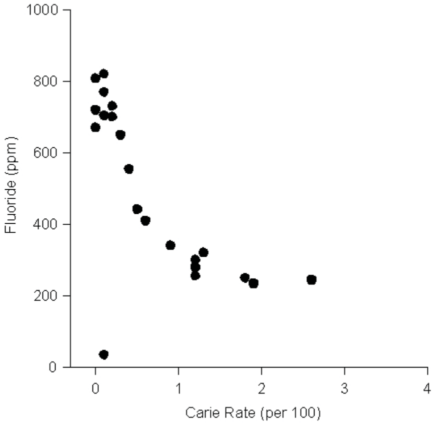
The chart below lists the life expectancy in years for people in the United States every five years from 1970 to 2005. What would you expect the life expectancy of a person in the United States to be in 2010, 2015, and 2020 based upon this data? Explain how you determined your values.

Date	1970	1975	1980	1985	1990	1995	2000	2005
Life Expectancy (in years)	70.8	72.6	73.7	74.7	75.4	75.8	76.8	77.4

Solution: There is a positive association.

Students recognize that points may be away from the other points (outliers) and have an effect on the linear model.
NOTE: Use of the formula to identify outliers is **not** expected at this level.

Students recognize that not all data will have a linear association. Some associations will be non-linear as in the example below:



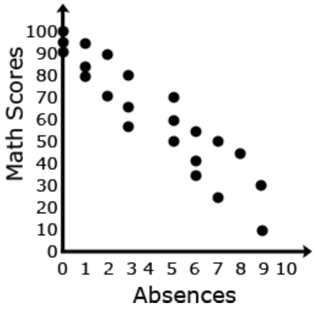
<table>
<tr>
<td>8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line</td>
<td>8.SP.2 Students understand that a straight line can represent a scatter plot with linear association. The most appropriate linear model is the line that comes closest to most data points. The use of linear regression is not expected. If there is a linear relationship, students draw a linear model. Given a linear model, students write an equation.</td>
</tr>
<tr>
<td>8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. <i>For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.</i></td>
<td>

8.SP.3 Linear models can be represented with a linear equation. Students interpret the slope and <i>y</i>-intercept of the line in the context of the problem.

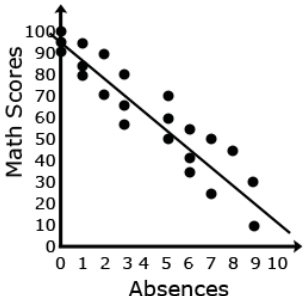
<u>Example 1:</u>

1. Given data from students’ math scores and absences, make a scatterplot.

<table>
<tr><th>Absences</th><th>Math Scores</th></tr>
<tr><td>3</td><td>65</td></tr>
<tr><td>5</td><td>50</td></tr>
<tr><td>1</td><td>95</td></tr>
<tr><td>1</td><td>85</td></tr>
<tr><td>3</td><td>80</td></tr>
<tr><td>6</td><td>34</td></tr>
<tr><td>5</td><td>70</td></tr>
<tr><td>3</td><td>56</td></tr>
<tr><td>0</td><td>100</td></tr>
<tr><td>7</td><td>24</td></tr>
<tr><td>8</td><td>45</td></tr>
<tr><td>2</td><td>71</td></tr>
<tr><td>9</td><td>30</td></tr>
<tr><td>0</td><td>95</td></tr>
<tr><td>6</td><td>55</td></tr>
<tr><td>6</td><td>42</td></tr>
<tr><td>2</td><td>90</td></tr>
<tr><td>0</td><td>92</td></tr>
<tr><td>5</td><td>60</td></tr>
<tr><td>7</td><td>50</td></tr>
<tr><td>9</td><td>10</td></tr>
<tr><td>1</td><td>80</td></tr>
</table>



2. Draw a linear model paying attention to the closeness of the data points on either side of the line.



3. From the linear model, determine an approximate linear equation that models the given data (about $y = -\frac{25}{3}x + 95$)

4. Students should recognize that 95 represents the <i>y</i>-intercept and $-\frac{25}{3}$ represents the slope of the line. In the context of the problem, the <i>y</i>-intercept represents the math score a student with 0 absences could expect. The slope indicates that the math scores decreased 25 points for every 3 absences.

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<tr>
<td></td>
<td>5. Students can use this linear model to solve problems. For example, through substitution, they can use the equation to determine that a student with 4 absences should expect to receive a math score of about 62. They can then compare this value to their line.</td>
</tr>
<tr>
<td>8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. <i>For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?</i></td>
<td>8.SP.4 Students understand that a two-way table provides a way to organize data between two categorical variables. Data for both categories needs to be collected from each subject. Students calculate the relative frequencies to describe associations.

<u>Example 1:</u>
Twenty-five students were surveyed and asked if they received an allowance and if they did chores. The table below summarizes their responses.

<table>
<tr><th></th><th>Receive Allowance</th><th>No Allowance</th></tr>
<tr><td>Do Chores</td><td>15</td><td>5</td></tr>
<tr><td>Do Not Do Chores</td><td>3</td><td>2</td></tr>
</table>

Of the students who do chores, what percent do not receive an allowance?
<i>Solution:</i> 5 of the 20 students who do chores do not receive an allowance, which is 25%</td>
</tr>
</table>

We would like to acknowledge the Arizona Department of Education for allowing us to use some of their examples and graphics.