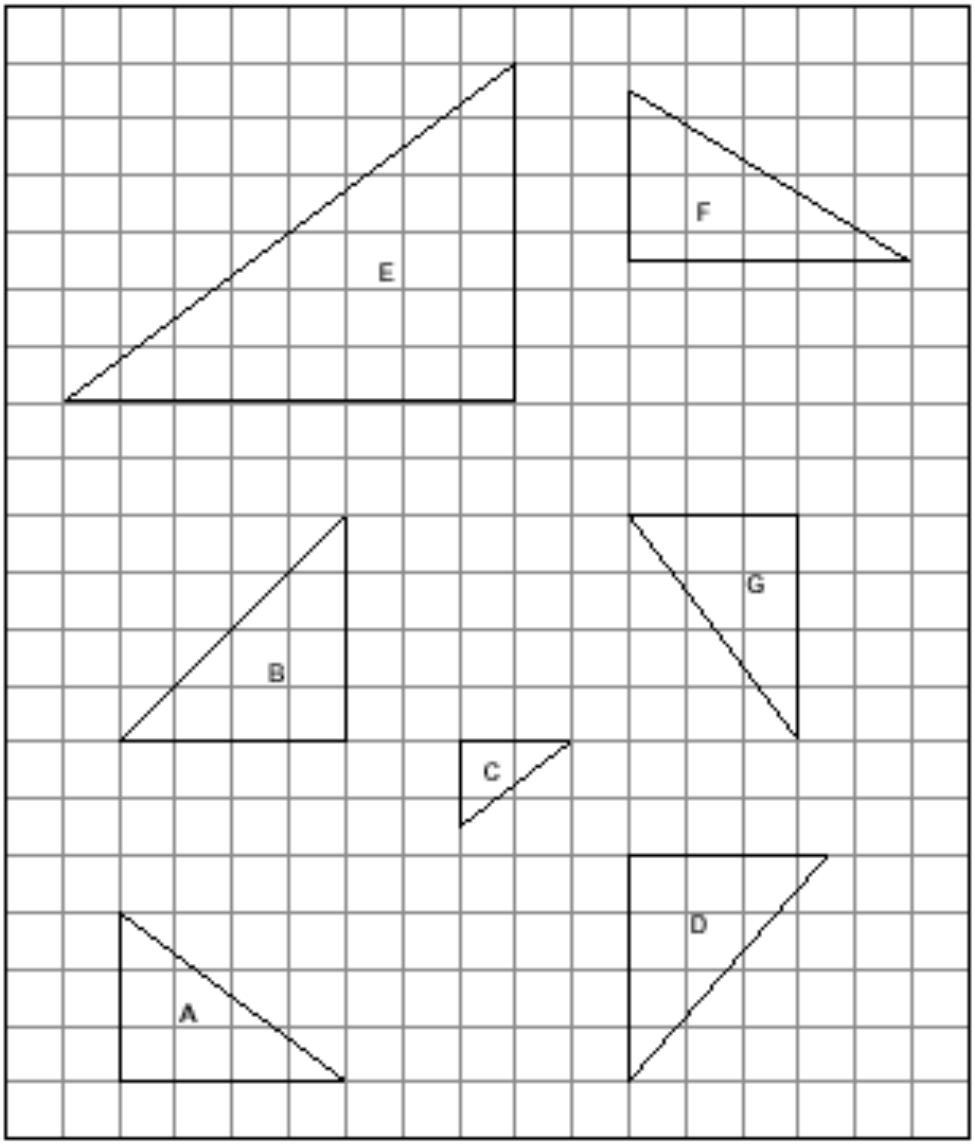
7th grade Task 3 Triangles

Student Task	Select triangles congruent and similar to a given right triangle on a grid. Explain the reason for their selections. Determine the area of a triangle when it is enlarged by a scale factor.
Core Idea 3 Geometry and Measurement	**Apply congruence and similarity to analyze mathematical situations. Apply appropriate techniques, tools, and formulas to determine measurements.** • Understand relationships among the angles, side lengths, and the areas of similar objects • Solve problems involving similarity and scale factors. Using proportional reasoning

Triangles

This problem gives you the chance to:
- reason about similar figures and scale factor

Here are some right triangles.



1. Which of the triangles on the opposite page is congruent to triangle A? _____
Explain your reasoning.

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2. Which of the triangles on the opposite page are similar to triangle A? _____
Explain how you decided.

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3. If triangle A is enlarged by a scale factor of 3, what will be the area of the new triangle?
Show your work.

_____

B

Triangles

Test 7 Form A Rubric

The core elements of performance required by this task are:
- reason about similar figures and scale factor

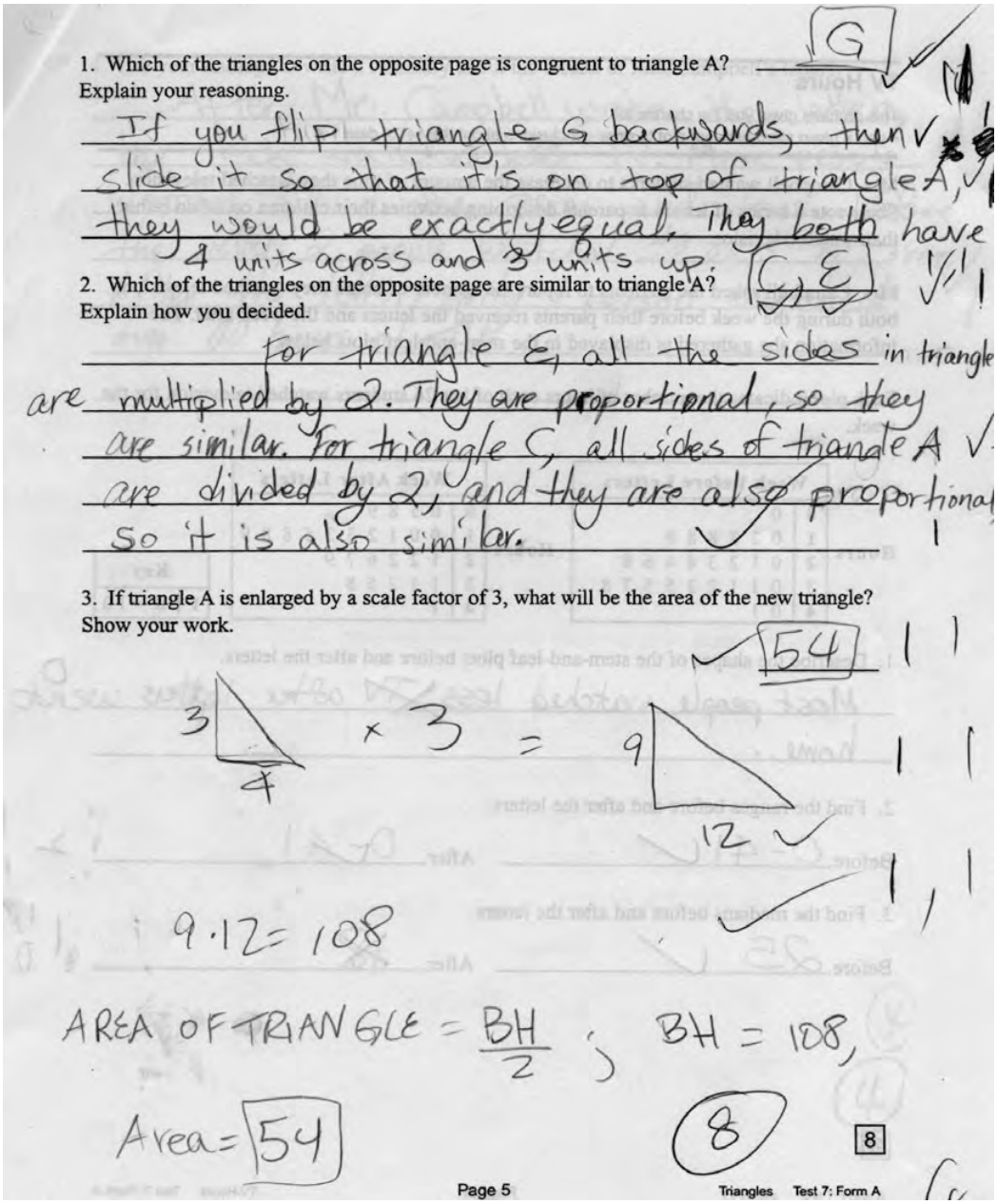
Based on these, credit for specific aspects of performance should be assigned as follows:

	Points	Section Points
1. Gives correct answer as:		
Triangle **G**	1	
Gives correct explanation such as:		
Both triangles have sides 3, 4, and a right angle between these two sides.	1	2
2. Gives correct answers as:		
Triangles **C and E** *(accept G)*	1, 1	
Gives correct explanation such as:		
All the sides of triangle C are half as long as the sides of triangle A. **or** All the sides of triangle E are twice as long as the sides of triangle A. **or** All the sides of triangle G are the same length as the sides of triangle A.	1 **or** 1 **or** 1	3
3. Gives correct answer as:		
54 square units	1	
Shows correct work such as:		
The area of triangle A is 6 square units.	1	
The area of the enlarged triangle is 9×6. **or** The height and base of the enlarged triangle are 12 and 9 units.	1 **or** 1	
The area of the enlarged triangle is $\frac{1}{2} \times 12 \times 9$.	1	3
Total Points		**8**

Looking at Student Work - Triangles

Few students could complete this task successfully. Student #1 shows an ability to quantify the length of the sides to prove congruency. The student offers the scale factor to prove why C and E are similar to A and can use scale factors to enlarge triangle A.

Student #1



1. Which of the triangles on the opposite page is congruent to triangle A? G
Explain your reasoning.

If you flip triangle G backwards, then slide it so that it's on top of triangle A, they would be exactly equal. They both have 4 units across and 3 units up.

2. Which of the triangles on the opposite page are similar to triangle A? C, E
Explain how you decided.

For triangle E, all the sides in triangle are multiplied by 2. They are proportional, so they are similar. For triangle C, all sides of triangle A are divided by 2, and they are also proportional so it is also similar.

3. If triangle A is enlarged by a scale factor of 3, what will be the area of the new triangle?
Show your work.

54

3 × 3 = 9

4 12

9 · 12 = 108

AREA OF TRIANGLE = $\frac{BH}{2}$; BH = 108,

Area = 54

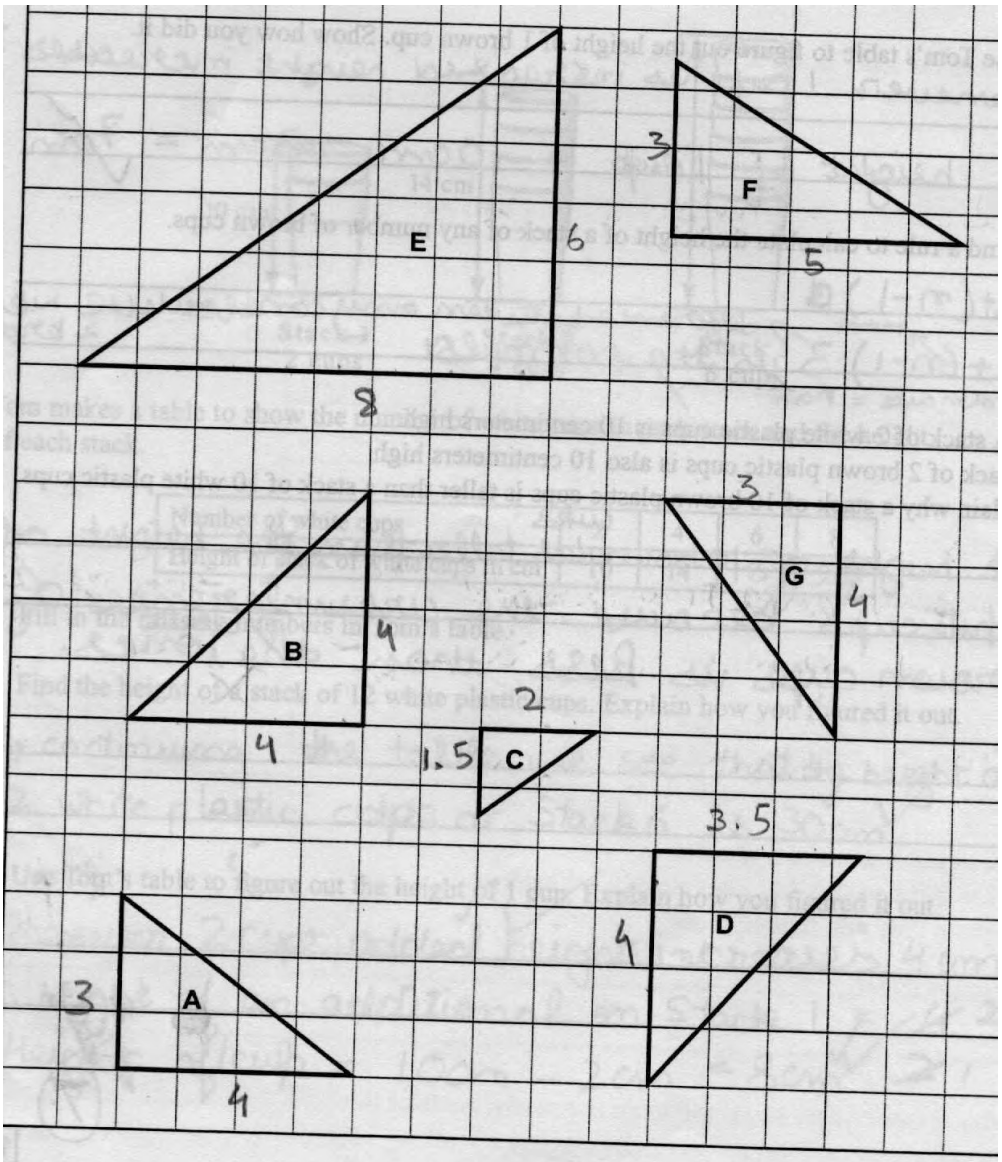
8

8

Page 5 Triangles Test 7: Form A

An important mathematical habit of mind is to look for relationships between sets of numbers and be able to quantify those relationships. Few students showed evidence of measuring the sides of the triangles on the first page of the task. Student #2 has found the dimensions of the sides and used that to successfully find congruent and similar triangles. The student does not quantify the scale factor for the similar triangles.

Student #2

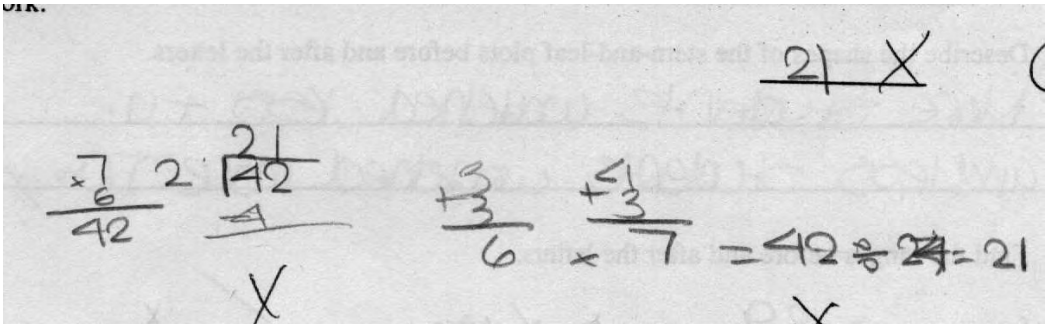


Explain how you decided.

Similar shapes have proportional sides. I see only A, C, and E triangles are similar, ✓

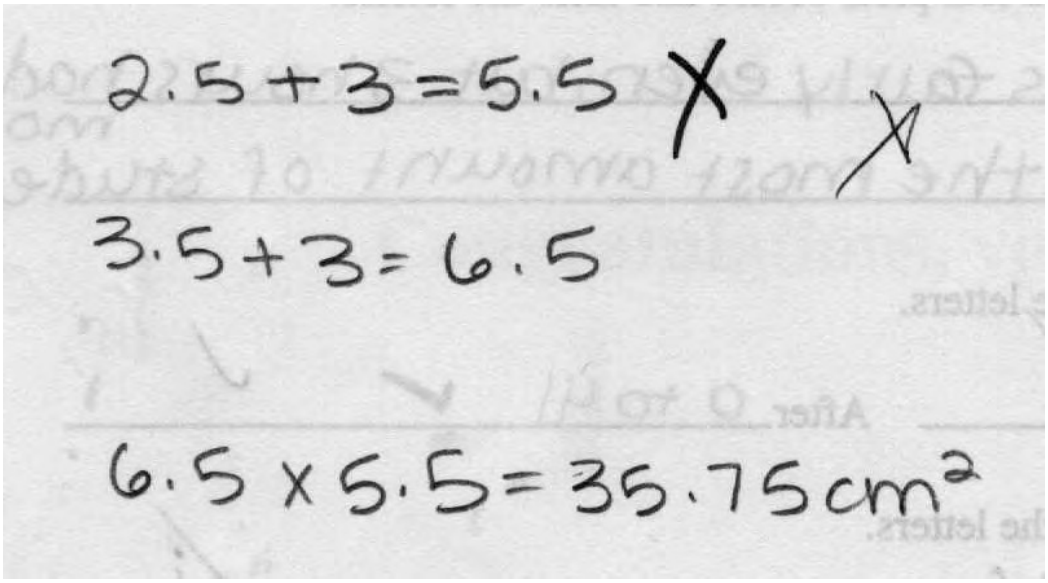
Many students do not understand the concept of scale factor. They use addition instead of multiplication to enlarge the triangle. Look at Student #3 below.

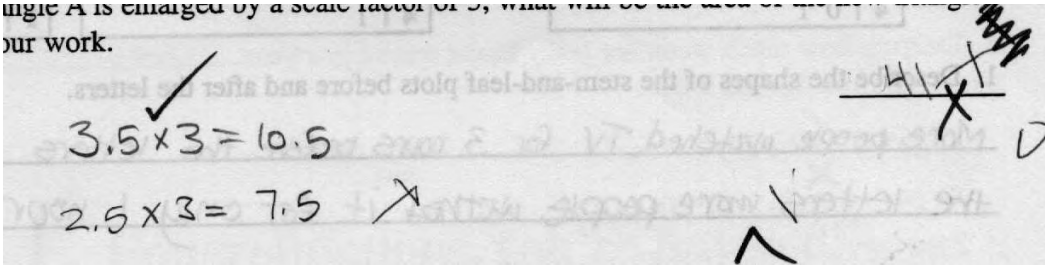
Student #3



Some students do not understand how to measure the sides of the triangle. Instead of counting lengths, they count the number of squares on a side. See Student #4's work.

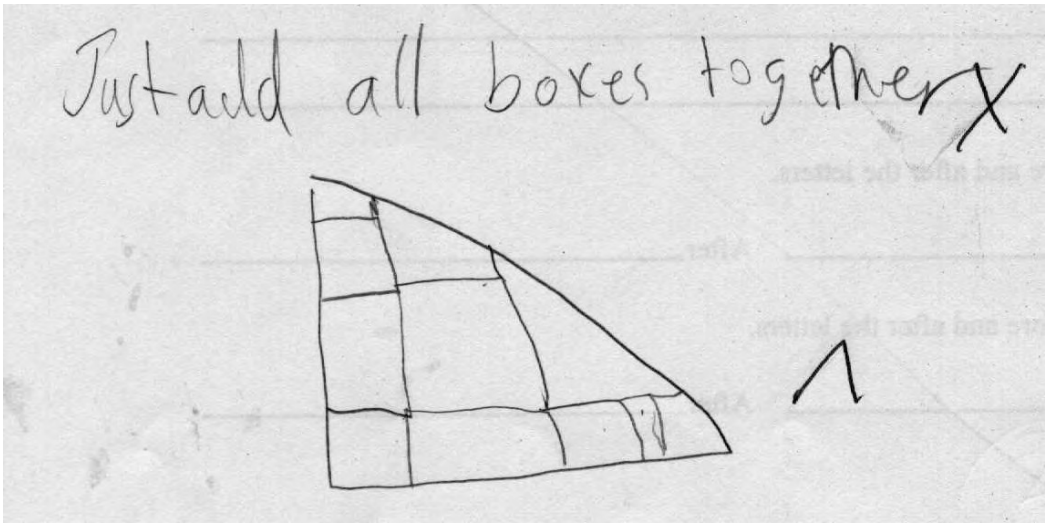
Student #4





Many students are still relying on drawing squares and counting to find area of a triangle. An example is the work by Student #5.

Student #5



Teacher notes: _____

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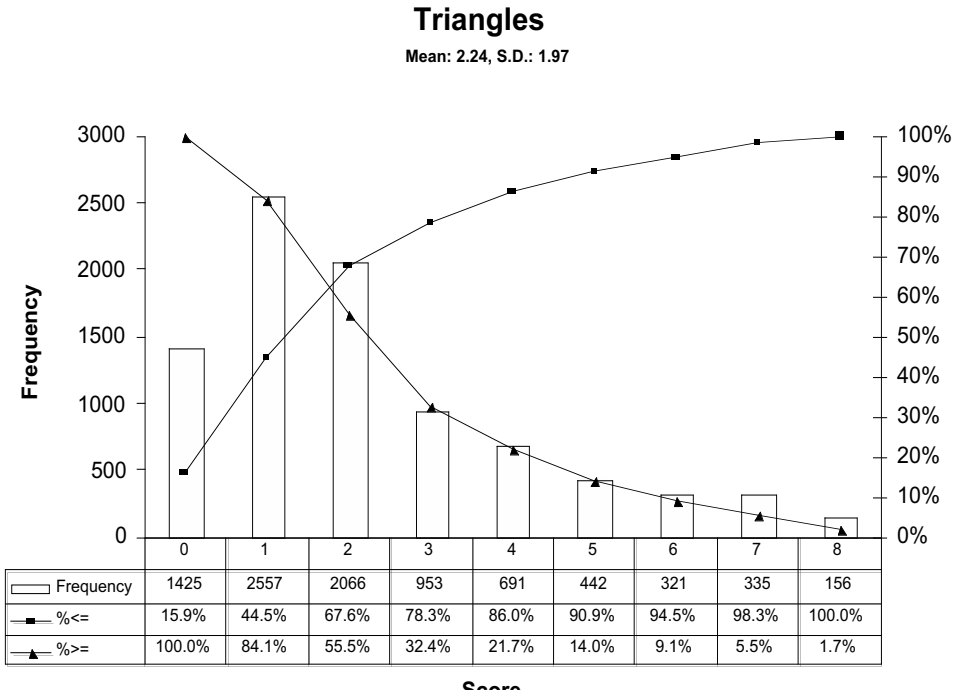

	0	1	2	3	4	5	6	7	8
Frequency	1425	2557	2066	953	691	442	321	335	156
%<=	15.9%	44.5%	67.6%	78.3%	86.0%	90.9%	94.5%	98.3%	100.0%
%>=	100.0%	84.1%	55.5%	32.4%	21.7%	14.0%	9.1%	5.5%	1.7%

The maximum score available for this task is 8 points.
The cut score for a level 3 response is 4 points.

Many students (84%) could correctly pick a congruent or similar triangle. About half the students could identify and explain which triangle is congruent and why or could pick a congruent and a similar triangle. About 30% of the students could pick similar triangles and either enlarge a triangel using scale factor or identify and explain why triangle G is congruent. Few students (9%) could explain similar triangles and use scale factor. Less than 2% of the students could meet all the demands of the task.

Triangles

Points	Understandings	Misconceptions
0	Most students tried the problem.	They were unclear on the definitions for congruent and similar. They had difficulty finding the lengths of the sides of the triangle, sometimes confusing length of side with number of squares on a side, therefore thinking the dimensions for triangle A were 2.5 by 3.5.
1	Students with this score could pick the correct triangle for question 1 or pick G as a similar triangle for question 2.	Students explanations showed a general lack of understanding of the definitions, often confusing similar and congruent. Many mentioned that shapes were facing the same way as a justification for congruent or similar. Triangles F and D were the most popular mistakes both for congruent and similar triangles.
2	About 1/3 of the students with this score could identify and justify why triangle G was congruent to A. Most students got answer 1 and picked G for a similar triangle, but did not correctly justify either answer.	Very few students tried to quantify the size of the triangles by giving measurements and those who did often used area instead of side or angle size. Many students are still trying to find area of a triangle in 3 by drawing and matching parts.
3	Many students start to pick E as the similar triangle. They can also pick the congruent triangle and explain it or give the dimensions for the new triangle in part 3.	Many students do not understand the multiplicative property of scale factor. They instead add the scale factor to the dimensions of the triangle, to get 7 and 6. Secondly many students do not realize that the product of base times height needs to be divided by 2.

6	Students with this score can identify G as congruent to A, pick E as a similar triangle and explain why, and use a scale factor to enlarge A.	Students have difficulty giving a complete explanation for why G is congruent. They try to rely on a definition, without using quantity to prove they have the same size and shape to support their argument for same size same shape. For students it is easier to find a similar triangle that is enlarged than one that is smaller.
8	Students with this score could give good mathematical explanations for similarity and congruency. They also could enlarge a triangle with a scale factor, and used a formula to find area of a triangle.	

Teacher notes: _____

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Based on teacher observation, these are the things seventh graders know and are able to do:

- Know a general definition for congruence.
- Use a variety of strategies to find area of a triangle.

Areas of difficulty for seventh graders, seventh graders struggle with:

- Constructing complete arguments about why two shapes are congruent.
- Understanding and proving similarity.
- Understanding and computing using scale factor.
- Using mathematical terminology when explaining their thinking; using terms like sides, angle, scale.
- Finding length of sides as part of the problem solving process, before searching for congruency and similarity. Knowing that quantifying size is important in identifying geometrical relationships.

Questions for Reflection - Triangles

- How often do students in your class need to develop a convincing argument? What types of weaknesses did you notice in their explanations?
- What are some recent classroom activities where they have made a mathematical justification?
- When you look at your student work, how many of your students showed evidence for measuring the sides of the triangles before working the problem? Did any of your students confuse number of squares on a side for the length of the side?
- Did your students understand scale factor or did they try to add three to the dimensions of triangle A?
- Are students fluent at finding area of a triangle? What is your evidence?
- What types of activities have students done this year with proportional reasoning?
- What do you think are the key geometrical concepts for your grade level?
- How much of your standard textbook material is devoted to using geometrical properties rather than applying and learning definitions?
- What do you think in-depth understanding of geometry should look like at this grade level?

Teacher notes: _____

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Implications for Instruction:

Students need to have a greater variety of geometrical experiences. Students have been finding area and perimeter on a grid since third grade. At seventh grade level student thinking around geometry needs to be pushed to a deeper level. Students should be comfortable with angles, angle measurement, scale factor and other geometrical attributes. Students need experiences where definitions are used to identify and quantify relationships, instead of learning definitions in isolation. All students need opportunities to solve problems in geometry. Students also need more work with proportional relationships. They do not see that scale factor is a multiplicative relationship rather than additive. A few students have not made the distinction between the squares on a side and the lengths of the line on a side. Teachers need to read student work to identify this type of error and give those students some individual instruction with measurement.

Teacher notes: _____

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